1. Use linear approximation for each of the following functions $f(x)$ and points a to approximate the corresponding numbers. Be sure to check the true value of $f(x)$ at $x = a$ (using a calculator), and compare it with the approximations.

   a. $f(x) = 8x^{3/2}, a = 1$, approximate $8 \cdot 1.1^{3/2}$.

   b. $f(x) = \mathrm{e}^{-x}, a = 0$, approximate $\mathrm{e}^{-0.3}$.

   c. $f(x) = (1 + x)^{-1}, a = 0$, approximate $\frac{1}{1.05}$.

2. a. Of all rectangles with a fixed perimeter of P, which one has the maximum area? (Give the dimensions in terms of P.)

   b. Of all rectangles with a fixed area A, which one has the minimum perimeter? (Give the dimensions in terms of A.)

   c. Find numbers x and y satisfying the equation $3x+y = 12$ such that the product of x and y is as large as possible.

3. Find the point P on the curve $y = x^2$ that is closest to the point $(18, 0)$. What is the least distance between P and $(18, 0)$? (*Hint: the minimum distance occurs at the same point as the minimum of the square of the distance.*)

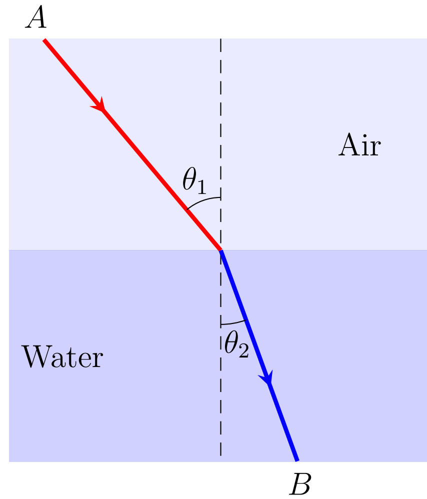
4. ($*$) An eight foot tall fence runs parallel to the wall of a house at a distance of five feet. Find the length of the shortest ladder that extends from the ground to the house without touching the fence. Assume that the vertical wall of the house is 20 feet high, and that the horizontal ground extends 20 feet from the house. (*You will need computing software to solve for the roots of one polynomial.*)

5. Find the dimensions of the right circular cylinder of maximum volume that can be placed inside a sphere of radius R.

6. Suppose you own a tour bus and you book groups of 20 to 70 people for a day tour. The cost per person is \$30 minus \$0.25 for every ticket sold. If gas and other miscellaneous costs are fixed at \$200, how many tickets should you sell to maximize your profit? Treat the number of tickets as a nonnegative integer.

7. ($*$) Suppose that a light source at A is in a medium in which light travels at speed v_1, and that the point B is in a medium in which light travels at speed v_2 (see figure, below). Using *Fermat's principle*, which states that light travels along the path that requires the minimum travel time, show that the path taken between points A and B satisfies $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$. (This

result is known as *Snell's law*.)



WeBWorK-style questions

8. Find $x > 0$ so that $x + \frac{1}{x}$ is as small as possible.

$$x = \quad .$$

9. A landscape architect wishes to enclose a rectangular garden of area $1000m^2$ on one side of length a by a brick wall costing $\$90/m$ and on the other three sides by a metal fence costing $\$30/m$.

   The total cost is $C = Xa + Y\frac{1}{a}$. Find X, Y.

$$X = \quad , Y = \quad .$$

   The cost is minimized for

$$a = \quad .$$

10. Consider a point $P = (a, b)$ with $a > 0, b > 0$ Consider a line of slope $m < 0$ going through P.

    The area of the triangle formed between the x-axis, y-axis and that line has area $A = Xm^Y(ma - b)^2$. Find X, Y.

$$X = \quad , Y = \quad .$$

    The derivative of the area can be written $\frac{\mathrm{d}A}{\mathrm{d}m} = \alpha m^\beta (am - b)(am + b)$. Find α, β.

$$\alpha = \quad , \beta, \quad .$$

    Assume that the area is maximized by $m = -2$, and this maximal area is $A = 8$, find a and b.

$$a = \quad , b = \quad .$$