Rota, Gian-Carlo (*b.* Vigevano, Italy, 27 April 1932; *d.* Cambridge, Massachusetts, 18 April 1999), *mathematics, combinatorics.* Rota is widely regarded as the founder of modern combinatorics. He was the spearhead of a movement that transformed combinatorics from a lightly regarded bag of tricks to a unified and deep discipline with profound connections to other areas of mathematics.

Life history. Rota was the son of Giovanni Rota, a civil engineer and architect. Giovanni Rota was a prominent anti-facist who had to flee Italy in 1945 to escape Mussolini's death squads. The remarkable story of his family's escape is recounted by Gian-Carlo Rota's sister Ester Rota Gasperoni in the two books *Orage sur le Lac* and *L'Arbre des Capulies.* Rota ended up completing his secondary school education in Ecuador. As a result of his escape story Rota was fluent in English, Italian, Spanish, and French.

In 1950 Rota entered Princeton University and graduated *summa cum laude* in 1953. He then attended graduate school at Yale University, receiving a Master's Degree in Mathematics in 1954 and a Ph.D. in 1956 under the supervision of Jacob T. Schwartz. After graduating from Yale Rota married Teresa Rondón (whom he subsequently divorced in 1980) and received a Postdoctoral Reseach Fellowship from the Courant Institute at New York University. The next academic year Rota became a Benjamin Peirce Instructor at Harvard University and in 1959 accepted a position at the Massachusetts Institute of Technology. Except for a two year hiatus 1965–67 at Rockefeller University, Rota remained at M.I.T. for the rest of his career. His honors and achievements include the Colloquium Lectures of the American Mathematical Society (1998), election to the National Academy of Sciences (1982), the Leroy P. Steele Prize for Seminal Contribution to Research (1988), Vice-President of the American Mathematical Society (1995–1997), four honorary degrees, and the supervision of 42 Ph.D. students. He held numerous consulting positions, including a fruitful association with Los Alamos Scientific Laboratory officially beginning in 1966. He died unexpectedly in his sleep at his home in Cambridge on April 18, 1999.

The foundations of combinatorics. Rota was originally trained in functional analysis, and his early work was in this area. In the early 1960's he became interested in combinatorics, then a rather seedy and disreputable backwater of mathematics. Combinatorics is concerned with the arrangement of discrete objects and looks at such problems as the existence of an arrangement, the number or approximate number of arrangements, relations among the different arrangements, and the "optimal" arrangment according to given criteria. In general the definitions involved are easy to understand, and the arrangements have little (obvious) internal structure. (Think of a jigsaw puzzle.) For this reason combinatorics was not regarded by most mathematicians as a serious subject. Rota had the vision to realize that on the contrary combinatorics had tremendous potential for elucidating and extending other areas of mathematics. He was able to recognize intuitively many problems to which combinatorics could be unexpectedly applied. As a consequence, he was the founder of the movement that lifted the subject of combinatorics to its current position as a major branch of mathematics.

Hermann Weyl has described Cayley's development of invariant theory as "[coming] into existence somewhat like Minerva: a grown-up virgin, mailed in the shining armor of algebra, she sprang forth from Cayley's jovian head." A similar statement could be made about the work of Rota on the foundations of combinatorics. Though led into combinatorics by his work on functional analysis, Rota's work on combinatorics was from the beginning a completely fresh combination of innovation and synthesis. His first paper in this area, published in 1964, had the characteristically audacious title "On the Foundations of Combinatorial Theory, I. Theory of Möbius Functions." This paper was the first in a series of ten seminal *Foundations* papers that transformed the field of combinatorics.

The Möbius function of a partially ordered set, the subject of Rota's first *Foundations* papers, was originally introduced by Louis Weisner and later independently Philip Hall as a tool for obtaining inversion formulas. Rota realized that this rather specialized and arcane topic had tremendous potential to unify, clarify, and generalize many apparently disparate combinatorial topics, including the calculus of finite differences, the Principle of Inclusion-Exclusion, set partitions, generating functions, and matroid theory. This paper also inaugurated the burgeoning subject of *topological combinatorics*.

Subsequent papers in the *Foundations* series, with various coauthors (and two by other authors given Rota's imprimatur), developed a unified theory of generating functions, a theory of operator calculus, a rigorous foundation of the classical "umbral calculus," and a revival of the nineteenth century subject of invariant theory. *Foundations IV* and *V*, written jointly with Jay Goldman and George Andrews, foresaw what is now a thriving cottage industry within mathematics and mathematical physics—the theory of q-analogues (or in more stylish terminology, "quantum mathematics").

Invariant theory. Invariant theory remained a subject dear to Rota's heart for the remainder of his career. To oversimplify greatly, invariant theory is concerned with properties of mathematical objects, especially polynomials, that are preserved by certain transformations. For instance, the polynomial x^2 is preserved by substituting $-x$ for x (a misleadingly simple example). This preservation of properties is intimately related to an object's symmetry, a major theme in present-day mathematics and the physical sciences. Work of many nineteenth and twentieth century mathematicians had shown that invariant theory has deep connections with algebra and geometry. Much *ad hoc* combinatorics was involved in this work, but the combinatorial aspects of invariant theory had not been adequately developed or systematized.

Foundations IX (with Peter Doubilet and Joel Stein) was Rota's first of over twenty papers in invariant theory. This paper began the development of a simple and powerful method to extend Hermann Weyl's work on vector invariants of the classical groups to the characterstic p case. The basis for this work was a symbolism for the product of minors called *bitableaux*, together with a "straightening algorithm" to express any bitableau in terms of special ones called *standard*. This work later found a host of applications, such as the proof by Formanek and Procesi that the general linear group is geometrically reductive and the proof by Pommerening that certain rings of invariants are finitely-generated. The techniques of straightening were later applied to such topics as resolutions of determinantal ideals in characteristic p (Akin and Buchsbaum), the Robinson-Schensted-Knuth (RSK) algorithm (Leclerc and Thibon), and the development of the notion of an "algebra with straightening law" (De Concini and Eisenbud, and independently Baclawski).

In another direction related to invariant theory, Rota developed the classical "symbolic method" into a powerful tool for doing computations in invariant theory. He extended his methods to the "letterplace (super)algebra" (based on a suggestion by Richard Feynman) and to skew-symmetric tensors. Rota also showed that the use of superalgebras allows Capelli's method of auxiliary variables to be extended to deal with symmetry and skew-symmetry in a uniform way.

Other activities. Rota had many academic activities not directly related to his research. He had a passionate interest in phenomenology and regularly taught courses and wrote papers in this area. His philosophy courses were the most popular in that subject at M.I.T., though he taught outside the Department of Linguistics and Philosophy and received no teaching credit for his efforts. He was in general an extremely popular teacher and adviser of undergraduate and graduate students at M.I.T. This commitment to students was a major factor in his receiving in 1996 the James R. Killian Faculty Achievement Award at M.I.T. Rota was also a bon vivant who loved to entertain all with whom he came in contact; he was exceptionally generous not only with his pocketbook, but also with his time and his ideas.

Although English was not Rota's native language, he regularly wrote essays and reviews with a masterful ear for English and with a surpassing clarity, incisiveness, and wit. The subjects of these essays ranged from mathematics and philosophy to personal reminisces centering on the human aspects of mathematicians he had known. Many of these essays are collected in the books *Discrete Thoughts* (coauthored with Mark Kac and Jacob Schwartz) and *Indiscrete Thoughts.*

Rota's long and fruitful association with Los Alamos National Laboratory began after meeting Stan Ulam in 1964. Rota rapidly became a significant member of the Los Alamos community and was appointed Director's Office Fellow in 1971. He was involved in a wide range of activities including collaboration, lectures, and politicking. He developed many deep friendships, especially with Ulam, whose strengths and imperfections he understood perfectly. More details of Rota's association with Los Alamos may be found in the essay by Michael Waterman in the 2000 *Notices of the American Mathematical Society* article devoted to Rota.

Rota was a founding editor in 1966 of the *Journal of Combinatorial Theory*, the first journal devoted entirely to combinatorics. Now there are over a dozen such journals, attesting to the tremendous growth of the field since Rota entered it. In 1967 Rota took over the faltering Academic Press journal *Advances in Mathematics* and remained in charge of the journal until his death. He single-handedly built *Advances* into a leading research journal, known especially for its eclectic content. A popular feature of *Advances* was Rota's book reviews, which managed to convey the essence of a complicated mathematics book in a few trenchant sentences.

WORKS BY ROTA

Kac, M., Rota, G.-C., and Schwartz, J. T. *Discrete Thoughts.* Boston: Birkhäuser, 1986.

Kung, J. P. S., ed. *Gian-Carlo Rota on Combinatorics.* Boston: *Birkhäuser*, 1995. A collection of Rota's papers on combinatorics with a number of commentaries. Includes *Foundations* I, III, IV, VI.

Dhombres, J., Kung, J. P. S., and Starr, N., eds. *Gian-Carlo Rota on Analysis and Probability.* Boston: *Birkhäuser*, 2003. A collection of Rota's papers on analysis and probability theory with a number of commentaries.

Indiscrete Thoughts. Boston: Birkhäuser, 1997.

Crapo, H. H., Rota, G.-C. "On the foundations of combinatorial theory. II. Combinatorial geometries." *Studies in Appl. Math.* 49 (1970): 109–133.

Andrews, G. E. "On the foundations of combinatorial theory. V. Eulerian differential operators." *Studies in Appl. Math.* 50 (1971): 345–375.

Doubilet, P. "On the foundations of combinatorial theory. VII. Symmetric functions through the theory of distribution and occupancy." *Studies in Appl. Math.* 51 (1972): 377–396.

Rota, G.-C.; Kahaner, D., and Odlyzko, A. "On the foundations of combinatorial theory. VIII. Finite operator calculus." *J. Math. Anal. Appl.* 42 (1973): 684–760.

Doubilet, P., Rota, G.-C., and Stein, J. "On the foundations of combinatorial theory. IX. Combinatorial methods in invariant theory." *Studies in Appl. Math.* 53 (1974): 185–216.

Bonetti, F., Rota, G.-C., Senato, D., and Venezia, A. M. "On the foundation of combinatorial theory. X. A categorical setting for symmetric functions." *Studies in Appl. Math.* 86 (1992): 1–29.

WORKS ABOUT ROTA

"Gian-Carlo Rota (1932–1999)." *Notices American Mathematical Society* 47 (2000): 203–216. Several essays on Rota's accomplishments, including a discussion of all the *Foundations* papers by Stanley and a survey of Rota's work on invariant theory by Buchsbaum and Taylor.

Gasperoni, E. R. *Orage sur le lac*, L'Ecole des loisirs, 1995.

Gasperoni, E. R. *L'Arbre des Capulies*, L'Ecole des loisirs, 1996.