No Credit for Being First

I had the experience of being the only person in the world to know the answer to a famous question raised by Jean-Pierre Serre and Irving Kaplansky (independently), but I deservedly received absolutely no credit for answering this question. How did this situation come about? Let R be a local ring, that is, a noetherian commutative ring (with identity) with a unique maximal ideal $\mathfrak{m}$. Let $k = R/\mathfrak{m}$, called the *residue class field* of R. A *minimal free resolution* of k as an R-module is an exact sequence

$$\cdots \to R^{\beta_2} \to R^{\beta_1} \to R^{\beta_0} \to k \to 0,$$

of R-modules where the β_i's are as small as possible. (We define $R^0 = 0$, the R-module with a single element 0. Only for very special R, called *regular*, do we have some $\beta_i = 0$.) It is easy to see that there exists such a resolution (simultaneously minimizing all β_i), and that it is unique up to "change of basis." In particular $\beta_0 = 1$. In 1965 Kaplansky and Serre asked whether the power series $\sum_{i\geq 0} \beta_i x^i$ is rational, i.e., can be written as a quotient of two polynomials in the ring $\mathbb{R}[[x]]$ of formal power series over $\mathbb{R}$. In 1979 Jan-Erik Roos showed that the answer to this question would be negative if there existed a noncommutative graded algebra $A = A_0 \oplus A_1 \oplus \cdots$ over the field k whose *Hilbert series* $\sum_{i\geq 0}(\dim_k R_i)x^i$ was nonrational and which satisfied an additional condition.

David Anick was an M.I.T. graduate student who received his degree in 1980. In the spring of that year I saw him at a tea and asked what was in his thesis. He replied that some problems in homotopy theory led him to show that certain graded algebras had nonrational Hilbert series. I told him that if his algebras satisfied a certain condition (Roos' condition) then his result would have a very nice consequence. When I explained the condition to him, he said that his algebras did satisfy it. Thus at that time I was the only person in the world who knew that the answer to the Serre-Kaplansky question was negative! Of course this situation changed about one minute later when I told Anick about the work of Serre and Kaplansky. It turned out that Serre was visiting Harvard at the time, but Anick was too shy to approach him. Thus I had the honor of informing Serre about Anick's result. Later Anick left mathematics and is now a psychiatrist who practices in Cambridge.