ERRATA

for *Enumerative Combinatorics*, vol. 1, 2nd ed., 2012

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- page 5, line 9–. Insert "a" after "is".
- page 23, line 6. Insert "a" after "is".
- page 24, line 2. Change c_i to c_1.
- page 34, line 15. It should be stated that for this second way, $d+1$ cannot be inserted at the end even though $d \notin D(w)$.
- page 38, line 18. Change "does" to "do".
- page 40, line 15. The intent of the parenthetical comment was to write the summation as

$$\sum_{g(4)\geq g(6)} \sum_{g(6)\geq g(3)} \cdots \sum_{g(5)\geq 0} q^{g(4)+\cdots+g(5)}.$$

  We are actually first summing on $g(5) \geq 0$, then $g(1) \geq g(5)$, etc.

- page 40, line 4–. Insert $\sum_{m\geq 0}$ before $\sum_{w\in\mathfrak{S}_d}$.
- page 43, line 4. Change "relection" to "reflection".
- page 43, line 14. Change $i \geq 0$ to $i \geq 1$.
- page 45, line 1. Change w to $T(w)$.
- page 47, line 11. Change $n \geq 2$ to $n \geq 1$.
- page 49, lines 20–22. The two sentence beginning "Figure 1.10 ..." and "Let $f(n)$ denote ..." should be interchanged, since $f(n)$ is used in the first of these sentences but defined in the second.
- page 50 , line 4. Change "tree" to "trees".
- page 53, line 1–. Change n to $n-1$.
- page 54, line 8. Change "number" to "numbers".

- page 62, equation (1.78). Change $j \in \mathcal{S}_i$ to $j \in S_i$.
- page 62, line 13–. Change $\mathcal{S}_j$ to S_i.
- page 65, line 7. Change proposition to Proposition.
- page 66, line 13. Change $F(0, q)$ to $F(x, 0)$.
- page 68, line 17–. Change "k of the γ_i's to "k γ_i's".
- page 71, line 2–. Change "then then" to "then the".
- page 76, line 2. Change "x to $-x$" to "t to $-x$".
- page 77, line 5–. Change "$f(0), f(0)$" to "$f(0), f(1)$".
- page 85, Theorem 1.10.4, line 3. Change $\mathrm{GL}(n, q)$ to $\mathrm{GL}(m, q)$.
- page 85, Theorem 1.10.4, line 4. Change $|\lambda(f)| \cdot \deg(f)$ to $m = |\lambda(f)| \cdot \deg(f)$
- page 88, line 13–. Change v_{ij} to v_{1j}.
- page 98, line 7. Change "in the monograph" to "is the monograph".
- page 109, Exercise 35(b,c,d). For the history of these results, see MathOverflow 63561.
- page 112, Exercise 54(a), line 1. Change $A(2n+1, n+1)$ to $A(2n+1, n)$.
- page 112, Exercise 55, line 2. Delete ", respectively", since $\mathrm{maj}(w^r)$ is expressed in terms of both $\mathrm{des}(w)$ and $\mathrm{maj}(w)$.
- page 113, Exercise 1.58(a), line 2. Change "and" to "can".
- page 115, Exercise 70(b), line 3. Change $(1, 0)$ to $(0, 1)$, and change "i.e," to "i.e.,".
- page 120, Exercise 94, second bullet. Change "Then" to "The".
- page 121, Exercise 96(c), line 3. Change 4.4.1.1 to 4.1.1.
- page 123, Exercise 116(b), line 2. Change distribuiton to distribution.

- page 126, line 2. Change $1 \leq i < j \leq n$ to "$1 \leq i \leq j \leq n$, excluding $(i, j) = (1, n)$,".
- page 126, Exercise 128(d), line 7. Change $\frac{2}{1+x}$ to $\frac{2x}{1+x}$.
- page 126, Exercise 129(a), line 3. Add $\sum_{k \geq 0}$ after $\sum_{n \geq 1}$.
- page 132, Exercise 160(c), lines 2,3. Change ξ to ζ.
- page 136, line 1–. Change q^{n-1} to q^{k-1}.
- page 137, Exercise 184(a), line 5. Change "polynomials $f(x) = \beta$ for $0 \neq \beta \in \mathbb{F}_q$" to "polynomial $f(x) = 1$".
- page 139, Exercise 193, line 2. Change γ_n to $\gamma(n)$.
- page 140, Exercise 199, line 4. The condition (only relevant in characteristic 2) should be that $A = -A^t$ and A has 0's on the main diagonal. Some people will take this to be the definition of "skew-symmetric," while others will call these matrices "alternating."
- page 140, Exercise 198(b), last displayed equation. Change (q^{2m-1}) to $(q^{2m-1} - 1)$ (twice).
- page 148, Exercise 56, line 3. Change $=$ to $\subseteq$.
- page 150, Exercise 22(b), line 6. Change x^{2i+1} to x^{2i-1}.
- page 156, line 2. Change *Combinatorica* to *Combinatoria*.
- page 163, Exercise 80. For the convoluted history of this result, see R. Gilbert, A Fine rediscovery, *Amer. Math. Monthly* **122** (2015), 322–331. In particular, the result is actually first due to N. J. Fine in 1959.
- page 166, Exercise 91(d), line 4. Replace this line with

$$1 + \sum_{n \geq 1} (-1)^n (x^n - x^{-n-1}) q^{\binom{n}{2}}.$$

- page 166, Exercise 92. This identity is actually due to I. Gessel and D. Stanton, *Trans. Amer. Math. Soc.* **277** (1983), 173–201 (equations (7.13) and (7.15)).

- page 168, Exercise 104, line 1. Notation is confusing! Change $f(x)$ to $g(x)$.

- page 168, Exercise 104, line 2. Change $f(n)$ to $g(n)$.

- page 168, Exercise 104, line 4. Change the four lines with a left brace to

$$= \begin{cases} \frac{1}{4}\left(10^n + (-1)^k 2^{2k+1}\right), & n = 4k \\ \frac{1}{4}\left(10^n + (-1)^k 2^{2k+1}\right), & n = 4k+1 \\ \frac{1}{4}10^n, & n = 4k+2 \\ \frac{1}{4}\left(10^n + (-1)^k 2^{2k+2}\right), & n = 4k+3. \end{cases}$$

- page 169, Exercise 109(a), line 4. Change "is bijection" to "is a bijection".

- page 171, line 4. The index entries for these four names is missing.

- page 173, Exercise 129. Change the parts **a** and **b** to **b** and **c**.

- page 173, Exercise 134, line 2. Change the period after Petersen to a comma.

- page 181, Exercise 164, line 1. Change $x = 0$ to $y = 0$.

- page 182 Insert after "$x + p_2x^2 + \cdots$." the following sentence:

  Then we need to show that $p_{2n} \in I := \langle p_2, \ldots, p_{2n-1} \rangle$, the ideal of the polynomial ring $K[a_2, a_3, \ldots]$ generated by $p_1, \ldots, p_{2n-1}$.

  The sentence after this one should begin a new paragraph.

- page 183, Exercise 168. The parts **b.**, **c.**, **d.**, **e.**, **f.** should be **d.**, **f.**, **g.**, **i.**, **j.**, respectively.

- page 186, Exercise 164. Change the parts **a** and **b** to **b** and **c**.

- page 186, Exercise 185, line 6. Insert $\prod_{d \geq 1}$ after $=$.

- page 190, Exercise 1.190(b), line 3. Replace this line with

$$\omega^*(n, q) = q^n - q^m - q^{m-1} - q^{m-2} - \cdots - q^{\lfloor n/3 \rfloor} + O(q^{\lfloor n/3 \rfloor - 1}).$$

- page 195, line 9–. Change "example" to "examples".
- page 196, equation (2.5). Change $Y \supseteq T$ to $Y \supseteq \emptyset$ or to just Y.
- page 198, line 2. This line should read:

  function $f_=$ satisfies $f_=(T) = f_=(T')$ whenever $\#T = \#T'$. Thus also $f_\geq(T)$

- page 199, line 11–. Change "i set" to "i-set".
- page 199, line 3–. Change $S - T$ to $S_n - T$.
- page 200, line 8. Change $j \geq i$ to $j \leq i$.
- page 203, line 6. Insert) before }.
- page 203, line 13. Change "nonatttacking" to "nonattacking".
- page 203, line 4–. Change "is" to "in".
- page 205, line 2–. Change $n - k - 1$ to $n - k + 1$.
- page 206, line 9. Change "suggest" to "suggests".
- page 207, line 4–. Change $x - s_1$ to $x + s_1$.
- page 208, line 10–. Change "a_3's" to "a_3 3's".
- page 210, line 9. Insert "one for 121," after "one for 112,".
- page 210, line 9–. Insert = between $d(n)$ and $\#D_n$.
- page 212, line 9–. Change $T \subseteq S$ to $Y \subseteq S$.
- page 216, line 5–. Insert) after x_{γ_i}.
- page 217, l. 14–. Change v to (x, y).
- page 217, l. 12–. Change v to (x, y).
- page 222, Exercise 10(b), lines 1 and 2. Change $E(n)/n!$ to $E(n)/n^n$ (twice).

- page 227, line 3–. On the left-hand side of the formula, remove one $\sum_{i\geq 0}$ and change y^i to t^i.
- page 229, Exercise 29(a), line 4. Change "partitions" to "compositions".
- page 235, Exercise 10(b), line 9. Change $E(n)/n!$ to $E(n)/n^n$.
- page 237, Exercise 19. Change $\binom{2n-k}{k}$ to $\frac{2n}{2n-k}\binom{2n-k}{k}$.
- page 237, line 2–. Change second a_{n-k+1} to a_{n-k+2}.
- page 248, line 5–. Change "an" to "a".
- page 249, Proposition 3.3.2, line 1. Change "condtions" to "conditions".
- page 251, line 1–. Change $\mathcal{I}$ to S.
- page 252, line 3. Insert "simple" before "matroids".
- page 253, line 9–. In the definition of finitary distributive lattice, add the condition that L has finitely many elements of each rank. (Thus for instance the distributive lattice of all finite subsets of an infinite set is locally finite with $\hat{0}$ but not finitary.)
- page 277, line 12–. Change $\dim(W \cup W')$ to $\dim(W + W')$.
- page 283, lines 8– to 6–. Delete the sentence "Let $\Lambda_t = \{s \in L(\mathcal{A})\colon s \leq t\}$, the principal order ideal generated by t."
- page 284, line 3–. Delete "real" at the end of the line.
- page 284, line 2–. Insert "over K" after "arrangements".
- page 285, line 9. Delete "$\#\mathcal{B}_1 = \#\mathcal{B} - 1$ and".
- page 285, line 11. Under the second Σ, change $\mathcal{B}_1 \in \mathcal{A}''$ to $\mathcal{B}_1 \subseteq \mathcal{A}''$
- page 291, line 10–. Change "sketch that" to "reference for".
- page 303, Theorem 3.15.8. To be completely accurate, one should assume that $P \neq \emptyset$.

- page 308, line 9. Change "an" to "and".
- page 313, Figure 3.33. The poset P_5 is drawn incorrectly.
- page 315, line 7. Change $a_s - a_{s+1}$ to $a_s - a_{s-1}$ (twice after the brace).
- page 317, after (3.80). The phrase "(since intervals of Eulerian posets are Eulerian)" is unnecessary since the formulas under consideration hold for any graded poset with $\hat{0}$ and $\hat{1}$.
- page 319, line 12–. Change $Q_1 * Q_2 * \cdots * Q_r$ to $R_1 * R_2 * \cdots * R_r$ (since Q_i already has been given another meaning).
- page 319, line 11–. Change the first Q_i to R_i.
- page 332, line 1–. Change γ_{j-1} to γ_{j-1}^2.
- page 332, bottom of page. Add the following paragraph:

  The proof that $(\gamma_j^*)^2 = 1$ is completely analogous. It also follows from the fact that $\gamma_j^2 = 1$, since we can assume without loss of generality that $j = p - 1$ and then apply the automorphism of G that sends each τ_k to τ_{p-k}.
- page 334, line 4. Change $z\delta$ to $z\delta_{p-1}$.
- page 336, line 11. Change $\varphi\colon \widehat{KP} \to \widehat{KP}$ to $\varphi\colon KP \to \widehat{KP}$.
- page 337, lines 14– to 11–. The sentence "For the algebraically minded ... formal power series." is not correct. We first need to consider non-commutative *polynomials* (not power series) and then pass to suitable completions.
- page 342, line 6. Change $(1-q)^2$ to $(1-q)$ (in the denominator).
- page 345, line 4. Change $p_{s-1} + p_s$ to $p_{j-1} + p_j$.
- page 345, line 13. Change A to $\boldsymbol{A}$ (boldface).
- p. 356, Exercise 15(g), line 3. Change 9655 to 11586.
- page 356, Exercise 16(b). Change the difficulty rating to [3] (or maybe [3+]). See M. Guay-Paquet, A. H. Morales, and E. Rowland, DMTCS proc. **AS** (FPSAC 2013 Paris) (2013), 253–264.

- page 360, Exercise 38. This exercise is incorrect for $1 < k < m$, where m is the maximum size of an antichain of P. A counterexample is given by $k = 2$ and $L = J(P)$, where $P = \mathbf{1} + (\mathbf{1} \oplus (\mathbf{1} + \mathbf{1}))$. It is true that $\#P_k = \#R_k$, even for modular lattices. See Exercise 3.101(d).

- page 360, Exercise 41(c,d). It is assumed that L is distributive.

- page 364, Exercise 55(b). The rating should be changed to [3–]. An exceptionally elegant proof was given by G. Stachowiak, *Order* **5** (1988), 257–259. Another elegant proof was given by B. Iriarte, `arXiv:1405.4880`. Iriarte proves the more general result that if G is the comparability graph of a poset P, then the number of linear extensions of $(G, \mathfrak{o})$ is maximized when $\mathfrak{o}$ respects the order P.

- page 374, Exercise 95. The stated identity is actually valid for the *dual* Möbius algebra, where one uses the join operation rather than the meet. The correct identity using the text definition of Möbius algebra is

$$\sum_{t \in L} \mu(t, \hat{1}) t = \left(\sum_{u \geq z} \mu(u, \hat{1}) u \right) \cdot \left(\sum_{v \vee z = \hat{1}} \mu(v, \hat{1}) v \right).$$

- page 375, Exercise 98, line 3. Change $\leq$ to $\geq$.

- page 378, Exercise 114(a). The answer should be

$$(x-1)(x-2)\cdots(x-n).$$

- page 379, Exercise 115(c), line 1. Change "0,1" to 0. (There are $\binom{n}{2}$ hyperplanes.)

- page 379, Exercise 116. The subscripts should not be in boldface.

- page 387, Exercise 3.139. The right-hand side of the displayed formula should be $2x(1+x)^{n-2}/(1-x)^n$.

- page 390, Exercise 158(a), line 4. Change **a.** at beginning of line to **i.**.

- page 390, Exercise 158(a), line 5. Change **b.** at beginning of line to **ii.**.

- page 390, Exercise 158(c). Delete one of the periods at the end of the line.

- page 390, Exercise 158(d), line 2. Change "thats" to "that".
- page 403, Exercise 189. NOTE. The result is also true for d odd, but the proof is quite a bit easier.
- page 404, Exercise 198. It should be mentioned that butterfly posets are also called "ladder posets," since there is an index entry for "poset, ladder."
- page 405, line 8–. The notation f_{00} is not defined until the next sentence.
- page 406, line 1–. Change $\mathrm{des}(w)$ to $\mathrm{asc}(w)$.
- page 407, line 1. Change $\mathrm{des}(w)$ to $\mathrm{asc}(w)$, and change "descents" to "ascents".
- page 407, line 3. Change $z\sum_{n\geq 1}$ to $\sum_{n\geq 1}$ (twice).
- page 407, line 5. Change $(x-1)^{n-1}$ to $(z-1)^{n-1}$.
- page 407, Exercise 205(b), line 2. Change 44605 to 44606 (private communication from Patrick Byrnes, 21 February 2012). Byrnes originally assumed that a vertex v could be covered by at most one singleton (element covering only v), but there is exactly one example up to rank 9 where this property need not hold. Byrnes also computes that there are 29,199,636 1-differential posets up to rank 10.
- page 408, Exercise 211, line 3. Change the numerator $r^2+(r+1)q-q^2$ to $r^2+r(r+1)q-rq^2$.
- page 408, Exercise 215(c). Change the difficulty rating [5] to [3].
- page 415, Exercise 36(b). This example is incorrect. For instance, $\{1,2,3\}$ and $\{1,2,4,5\}$ have no meet.
- page 426, Exercise 88. Another solution is the following. We have

$$\sum_{s\leq t}\mu(s,t)=\sum_{s}\sum_{t\geq s}\mu(s,t).$$

  By the definition (3.15) of μ, this latter sum over t is 0 if $t\neq\hat{1}$ and is 1 if $t=\hat{1}$, and the proof follows. Note that this proof only requires P

to have a $\hat{1}$. More generally, we leave the reader to show that for *any* finite poset P, $\sum_{s \le t} \mu(s,t) = 1 + \mu_{\widehat{P}}(\hat{0}, \hat{1})$, where $\widehat{P}$ denotes P with a $\hat{0}$ and $\hat{1}$ adjoined.

- page 431, line 2–. Change **23** to **33**.
- page 449, Exercise 163(a), line 3. Change $2e_P(p-1) - (p-1)e(p)$ to $(2e_P(p-1) - (p-1)e(P))/p!$.
- page 458, Exercise 185. Part **f.** should be **k.**, and **g.** should be **l.**
- page 459, line 5–. Should be $\beta_P(4,5,6) = -1$.
- page 465, line 7–. Change "degree j" to "degree $j-1$".
- page 468, Proposition 4.2.2, line 5. Change the period after E_f to a comma.
- page 469, line 6–. Change $\sum_{n \ge 0} f(n)$ to $\sum_{n \ge 0} f(n)x^n$.
- page 511, line 18–. Change (v, b_i) to $(v, i),\ 1 \le i \le k$.
- page 522, Figure 4.29. The vertex labels should be 0 and 1, not 00 and 10.
- page 528, Exercise 4.2, line 3. Change "over" to "in".
- page 530, Exercise 4.12. Change $0.00010203050813213455\cdots$ to $0.000101020305081321345 5\cdots$, and change $1, 2, 3$ to $1, 1, 2, 3$.
- page 530, Exercise 4.14, line 2. Insert "relatively prime" after "choose a, b".
- page 531, Exercise 22, line 1. Change $\alpha \in \mathbb{C}$ to $0 \ne \alpha \in \mathbb{C}$.
- page 532, Exercise 25(e), line 3. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 532, Exercise 25(e), line 5. Change $N_1(m)$ to $N_1(q^m - 1)$ and $N_{-1}(m)$ to $N_{-1}(q^m - 1)$.
- page 532, Exercise 25(e), line 8. Change $N_1(m)$ to $N(2^m - 1)$.
- page 532, Exercise 25(e), line 10. Change $N_1(m)$ to $N_1(2^m - 1)$.

- page 533, line 2. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 533, line 5. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 533, line 7. Change $N_1(m)$ to $N_1(3^m - 1)$.
- page 533, line 8. Change $N_2(m)$ to $N_2(3^m - 1)$.
- page 533, line 10. Change $N_1(m)$ to $N_1(3^m - 1)$.
- page 533, line 11. Change $N_2(m)$ to $N_2(3^m - 1)$.
- page 539, Exercise 47, line 1–. Change 1.1.8.6 to 1.8.6.
- page 542, line 4. Change $n \times n$ to $(n+1) \times (n+1)$.
- page 542, line 10. Delete) after u_6.
- page 546, Exercise 4.75(d). Change the difficulty rating to [3–]. A combinatorial proof (though not a simple bijection) was found by Jacob Scott, private communication dated September 2, 2012.
- page 548, Exercise 2(a), line 6. The assertion "Clearly, we can write $f(x) = P(x)/Q(x)$ for some relatively prime integer polynomials P and Q." is not so clear. One must show that if $F(x) \in \mathbb{Q}[[x]]$ and $F(x) = R(x)/S(x)$ where $R, S \in \mathbb{C}[x]$, then one can write $F(x) = P(x)/Q(x)$ where $P, Q \in \mathbb{Q}[x]$. This statement does have a fairly simple proof, which we leave as an exercise.
- page 549, line 1–. Change x^{m^2} to x^{n^2}.
- page 552, Exercise 14. The smallest known pair (a, b) seems to be

$$\begin{array}{rcrcl} a & = & 106276436867 & = & 31 \cdot 3128272157 \\ b & = & 35256392432 & = & 2^4 \cdot 2203524527 \end{array}\text{ ,}$$

  due to M. Vsemirnov, *J. Integer Seq.* **7** (2004), article 04.3.7.
- page 557, line 4–. Change x^ℓ to x^l.
- page 558, line 1. Delete comma after *Wochenschrift*.

- page 558, Exercise 43. The sequence $t(3), t(4), \ldots$ is known as *Alcuin's sequence*, after Alcuin of York (730s or 740s – 19 May 804). For a survey see D. J. Bindner and M. Erickson, *Amer. Math. Monthly* **119** (2012), 115–121.

- page 560, Exercise 46(b), line 1. Delete the first "many".

- page 560, Exercise 46(b), line 2. Change $f(n+1), f(n+2), \ldots, f(n+d)$ to "$(f(n+1), f(n+2), \ldots, f(n+d))$ for fixed d".

- page 564, Exercise 58(f), line 1. Change "order polynomials" to "Ehrhart polynomials".

- page 567, Exercise 75(c), line 1. Delete "directed".

- page 569, Exercise 80, displayed equation, line 1. Change $v_i' y$ to $v_i y$.

- page 570, Exercise 82(e,f). For the paper of J. Schneider, see *Electronic J. Combinatorics* **21** (2014), P1.43; `arXiv:1206.6174`.

- page 571, line 3. Change "chose" to "chosen".

- page 578. The following item is missing: First edition—Supplementary Exercise 1.7 = Second edition—Exercise 1.114(b).

- page 579. The following three items are missing: First edition—Supplementary Exercise 3.19 = Second edition—Exercise 3.44; First edition—Supplementary Exercise 3.10 = Second edition—Exercise 3.63; and First edition—Supplementary Exercise 3.17 = Second edition—Exercise 3.189.

- page 620, Stembridge entry, line 6. Change $q = 1$ to $q = -1$.