

# EC2 SUPPLEMENT: PAPERBACK EDITION OF 2001

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Here I will maintain supplementary material for *Enumerative Combinatorics*, volume 2 (paperback edition of 2001). This will include errata, updated references, and new material. I will be continually updating this supplement.

**NOTE.** References to math.CO refer to the combinatorics section of the xxx Mathematics Archive at [xxx.lanl.gov/archive/math](http://xxx.lanl.gov/archive/math). A front end site for math.CO is [front.math.ucdavis.edu/math.CO](http://front.math.ucdavis.edu/math.CO).

- p. 6, line 10. Change situations to situations.
- p. 11, line 3. Change  $E_c(n)$  to  $E_c(x)$ .
- p. 18, line 3. Change  $(n)_2$  to  $n(n-2)$ .
- p. 20, line 9. Change  $Z(\mathfrak{S}_n)$  to  $\tilde{Z}(\mathfrak{S}_n)$ .
- p. 24, line 4 (after figure). Change  $\lim_{n \rightarrow \infty}$  to  $\lim_{k \rightarrow \infty}$ .
- p. 25, line 5. Change  $\subseteq$  to  $\in$ .
- p. 33, line 5-. Change  $\text{ord}(\tau_k)$  to  $\text{ord}(\tau_j)$ .
- p. 34, Lemma 5.3.9. Delete the first sentence, viz., “Let  $w \in \mathcal{A}^*$ .”
- p. 35, line 10. Change  $w \in \mathcal{B}^*$  to  $w \in \mathcal{B}_r^*$ .
- p. 35, line 8-. Change  $\mathbb{A}$  to  $\mathcal{A}$ .
- p. 36, lines 15–16. Change “beginning with a 1” to “ending with a  $-1$ ”.
- p. 36, line 1-. Insert  $+\cdots$  before  $=$ . (The left-hand side is an infinite sum.)

- p. 42, line 11. Change  $t^{r_1}$  to  $t_1^{r_1}$ .
- p. 51, line 9–. Change  $Q_i = \Pi_i^{(2)}$  to “when  $Q_i$  is given by Example 5.5.2(d) for  $r = 2$ ”.
- p. 59, line 9. Change “Since the rows” to “Since the columns”.
- p. 59, line 13. Change “Because the columns” to “Because the rows”.
- p. 62. Example 5.6.12, line 5. Change “modulo  $n$ ” to “modulo  $2^n$ ”.
- p. 63, line 12. Change “sequence” to “sequences”.
- p. 65, line 8. Change “Theorem” to “Lemma”.
- page 80, Exercise 5.21, line 1. The equation reference to the second edition of EC1 should be (4.41).
- p. 87, equation (5.111). We need to add the further condition that  $p_n(0) = \delta_{0n}$ . Otherwise, for instance, the polynomials  $p_n(x) = (1+x)^n$  satisfy (iv) with  $Q = \frac{d}{dx}$  but fail to satisfy (i)–(iii).
- p. 97, Exercise 5.51. It is not true that (ii) implies (i), e.g., when  $C(x) = c$ . One needs to add the hypothesis that  $[x]C(x) \neq 0$ , so that  $(C(x) - c)^{\langle -1 \rangle}$  exists. Substituting  $x C(B(x))$  for  $x$  in (ii) yields

$$x C(B(x)) / C(A(x C(B(x)))) = x,$$

so  $C(B(x)) = C(A(x C(B(x))))$ . Substituting  $B(x)^{\langle -1 \rangle}$  for  $x$  yields  $C(x) = C(A(B(x)^{\langle -1 \rangle} C(x)))$ . Subtract  $c$  from both sides and apply  $(C - c)^{\langle -1 \rangle}$  to get  $x = A(B(x)^{\langle -1 \rangle} C(x))$ . Applying  $A^{\langle -1 \rangle}$  to both sides gives (i). This argument is due to Daniel Giaimo and Amit Khetan and (independently) to Yumi Odama.

- p. 101, line 3. Change  $J_0[(2-t)/\sqrt{t-1}]$  to  $J_0(\sqrt{-t}(2-t)/(1-t))$ .
- p. 102, Exercise 5.71. It would be better not to specify the degree  $d$  of  $G$ , since (as stated in the solution)  $d = \lambda_1$ .
- p. 103, Exercise 5.74(d). Replace the first two sentences with: Show that all vertices have the same outdegree  $d$ . (By (c), all vertices then also have indegree  $d$ .)

- p. 103, Exercise 5.74(f). For further information, see F. Curtis, J. Drew, C.-K. Li and D. Prigel, *J. Combinatorial Theory (A)* **105** (2004), 35–50, and the references given there.
- p. 108, Exercise 5.7(a), line 7. Change  $b_{2n}$  to  $b_{2n-k}$ .
- p. 137, Exercise 5.45, line 1. Change  $kxy^k$  to  $(k+1)xy^k$ .
- p. 137, Exercise 5.45, line 4. Change this equation to

$$y = x + 2xy + 3xy^2 + \cdots = \frac{x}{(1-y)^2}.$$

- p. 139, Exercise 5.47(c), line 7. A direct combinatorial proof was given by M. Bousquet-Mélou and G. Schaeffer, *Advances in Applied Math.* **24** (2000), 337–368.
- p. 147, Third Solution. The first two lines should be: Equation (5.53) can be rewritten (after substituting  $n+k$  for  $n$ )

$$(n+k)[x^n] \frac{1}{k} \left( \frac{F^{(-1)}(x)}{x} \right)^k = [x^n] \left( \frac{x}{F(x)} \right)^{n+k}. \quad (5.140)$$

- p. 162, lines 13– to 12–. Change “Thus any algebraic power series, as defined in Definition 6.1.1” to “Thus any algebraic function, i.e, any solution  $\eta$  to (6.2)”.
- p. 175, line 1. Change  $\{9, 11\}$  to  $\{9, 14\}$ .
- p. 175, line 2. Change  $x^{11}$  to  $x^{14}$ .
- p. 175, line 4. Change  $v^{11}$  to  $v^{14}$ .
- p. 175, line 11. Change Theorem 5.4.1 to Theorem 5.4.2.
- p. 176, line 4–. Change  $(n+2)$ -gon to  $(n+1)$ -gon.
- p. 178, line 16. Change intesect to intersect.
- p. 190, Theorem 6.4.6. It should be assumed that  $\text{char } K = 0$ , since this assumption is used for (6.12). On the other hand, *every* power series over a field of characteristic  $p > 0$  is  $D$ -finite since its  $p$ th derivative is 0. See MathOverflow 420530.

- p. 192, line 9-. Change  $u(0) = 0$  to  $v(0) = 0$ .
- p. 192, lines 8- to 7-. The example  $v = \log(1 + x^2) - 1$  is confusing since  $v(0) \neq 0$ . Nevertheless the series  $u(v(x)) = \sqrt{\log(1 + x^2)}$  is well-defined formally since we can write

$$\sqrt{\log(1 + x^2)} = x \sqrt{\frac{\log(1 + x^2)}{x^2}}.$$

It would have been more accurate to define

$$v(x) = \frac{\log(1 + x^2)}{x^2} - 1.$$

The same remarks apply to Exercise 6.59.

- p. 212, line 1. The statement that Catalan number enumeration originated with Segner and Euler in 1760 (or actually 1758/59 in the cited references) is inaccurate. The enumeration of polygon dissections was stated by Euler in a letter to Goldbach in 1751. This letter is printed in P.-H. Fuss, *Correspondance Mathématique et Physique*, Tome. 1, Acad. Sci. St. Petersburg, 1843; reprinted in *The Sources of Science*, No. 35, Johnson Reprint Corporation, New York and London, 1968, pp. 549–552.
- p. 212, lines 16–17. I have forgotten the source for the statement that Netto was the first to use the term “Catalan number.” Can anyone provide a reference?
- p. 219, Exercise 6.16. A combinatorial proof was first given by R. Sulanke, *Electronic J. Combinatorics* **7**(1), R40, 2000. A sharper result was subsequently proved combinatorially by D. Callan, A uniformly distributed parameter on a class of lattice paths, math.CO/0310461.
- p. 221, Exercise 6.19(j). This problem appeared as Problem A5 on the 2003 William Lowell Putnam Mathematical Competition. Many participants found the following bijection with 6.19(i) (Dyck paths from  $(0, 0)$  to  $(2n, 0)$ ): Let  $D$  be a Dyck path from  $(0, 0)$  to  $(2n, 0)$ . If  $D$  has no maximal sequence of  $(1, -1)$  steps of even length ending on the  $x$ -axis, then just prepend the steps  $(1, 1)$  and  $(1, -1)$  to the beginning of  $D$ . Otherwise let  $R$  be the rightmost maximal sequence of  $(1, -1)$

steps of even length ending on the  $x$ -axis. Insert an extra  $(1, 1)$  step at the beginning of  $D$  and a  $(1, -1)$  step after  $R$ . This gives the desired bijection.

- p. 224, item **ii**, line 5. Change  $S(w) = w$  to  $S(w) = 12 \cdots n$ .
- p. 228, item **iii**, line 3. To be precise, the displayed sequences should have the initial and final 1's deleted.
- p. 230, Exercise 6.21(b), line 3. Change 5.3.11 to 5.3.12
- p. 230, Exercise 6.23. Brian Rothbach has pointed out that this problem can be given another stipulation: serieshelpmate in 20. By parity considerations (Black cannot lose an odd number of moves with a knight) the solution is the same as before except Black plays Pa7-a6 instead of Pa7-a5. Thus the number of solutions is  $C_{10} = 16796$ .
- p. 233, Exercise 6.30, line 3. It would be less ambiguous to change “this exercise” to “that exercise”.
- p. 235, Exercise 6.34, line 4. At the end of the line it should be mentioned that the polynomial  $g(L_n, q)$  of Exercise 3.71(f) is a further  $q$ -analogue of  $C_n$ . An additional reference for this polynomial is R. Stanley, *J. Amer. Math. Soc.* **5** (1992), 805–851 (Prop. 8.6).
- p. 236, Exercise 6.34(b,c). While (b) is correct as stated (in the paperback edition of 2001), it would be best to change  $q^n c_n(q)$  on line 6 to  $c_n(q)$  (as it was in the hardcover edition of 1999) and “nonnegative” on line 8 to “nonpositive”. In this way part (c) remains valid. If part (b) is kept as it is, then change  $c_n(t; q)$  in line 4 of part (c) to  $q^n c_n(t; q)$ .
- p. 238, Exercise 6.38(d), line 1. Change  $(n, n)$  to  $(n, 0)$ .
- p. 239, Exercise 6.39(h). A period is missing at the end of the sentence.
- p. 241, Exercise 6.41, line 1. Change  $S^2(w) = w$  to  $S^2(w) = 12 \cdots n$ .
- p. 247, Exercise 6.59. See the item above for p. 192, lines 8– to 7–.
- p. 250, Exercise 6.3, line 3. Replace “ $r = s + \frac{1}{2}$  for some  $s \in \mathbb{Z}$ ” with “ $r$  cannot be a negative integer”.

- p. 250, Exercise 6.3, paragraph 3. The earliest proof that  $\sum_{n \geq 0} \binom{2n}{n}^t x^n$  isn't algebraic for any  $t \in \mathbb{N}$ ,  $t > 1$ , appears in the paper P. Flajolet, *Theoretical Computer Science* **49** (1987), 283–309 (page 294). Flajolet shows that if  $\sum a_n x^n$  is algebraic and each  $a_n \in \mathbb{Q}$ , then  $a_n$  satisfies an asymptotic formula

$$a_n = \frac{\beta^n n^s}{\Gamma(s+1)} \sum_{i=0}^m C_i \omega_i^n + O(\beta^n n^t),$$

where  $s \in \mathbb{Q} - \{-1, -2, -3, \dots\}$ ,  $t < s$ ,  $\beta$  is a positive algebraic number, and the  $C_i$  and  $\omega_i$  are algebraic with  $|\omega_i| = 1$ . A simple application of Stirling's formula shows that if  $a_n = \binom{2n}{n}^t$ , then  $a_n$  does not have this asymptotic form when  $t \in \mathbb{N}$ ,  $t > 1$ .

- p. 256, Exercise 6.19(d), line 2. Delete “ignoring the root edge”.
- p. 257, Exercise 6.19(k). Update the reference to *J. Integer Seq.* **4** (2001), Article 01.1.3; available electronically at

<http://www.research.att.com/~njas/sequences/JIS>.

- p. 258, Exercise 6.19(s), line 1. Change  $a_i$  to  $a_i - 1$ .
- p. 260, Exercise 6.19(ee), lines 10–12. The statement that the first published proof of the enumeration of 321-avoiding permutations is due to D. G. Rogers is inaccurate. Knuth provided such a proof in 1973 in the reference given in the first paragraph of the solution. Moreover, a bijective proof was found by D. Rotem, *Inf. Proc. Letters* **4** (1975), 58–61.
- p. 274, line 2. Change “D. Vanquelin” to “B. Vauquelin”.
- p. 275, Exercise 6.40, line 6. Change M. O. J. to W. O. J.
- p. 278, Exercise 6.53, line 3. Change  $Q(x) = x - 2$  to  $Q(x) = -x - 2$ .
- p. 281, Exercise 6.60. An elegant proof based on Gröbner bases was given by Chris Hillar, *Proc. Amer. Math. Soc.* **132** (2004), 2693–2701.
- p. 282, Exercise 6.63(b), line 2. Change 1847 to 1848.

- p. 293, lines 11-13. Replace “, and such that the ... exist.)” with a period. (The deleted condition automatically holds.)
- p. 295, Figure 7-3. In the expansion of  $h_{41}$ , the coefficient of  $m_{41}$  should be 2.
- p. 298, line 10-. Change “if follows” to “it follows”.
- p. 301, line 7. Change 1.1.9(b) to 1.9(b).
- pp. 314–315, proof of Proposition 7.10.4. Change  $\lambda$  to  $\lambda/\mu$  throughout proof.
- p. 317, line 12-. Change “clearly impossible” to “clear”.
- p. 329, line 15-. Change  $x$ 's to  $X$ 's.
- p. 336, line 7 (counting the displayed tableau as a single line). Change 7.8.2(b) to 7.8.2(a).
- p. 346, line 3-. Change “forms a border strip” to “forms a border strip or is empty”.
- p. 346, line 1-. Change  $\lambda^i/\lambda^{i+1}$  to  $\lambda^{i+1}/\lambda^i$ .
- p. 348, line 9. Change  $\chi_\lambda(\mu)$  to  $\chi^\lambda(\mu)$ .
- p. 352, line 2 of proof of Proposition 7.18.1. Change  $\sum_{\mu} z_\lambda^{-1} f(\lambda) p_\mu$  to  $\sum_{\lambda} z_\lambda^{-1} f(\lambda) p_\lambda$ .
- p. 354, line 4. Change “in” to “is”.
- p. 354, line 5. Change “a integral” to “an integral”.
- p. 355, line 4. Add a period after “nonnegative”.
- p. 359, line 6. Change the subscript  $\alpha_S$  to  $\text{co}(S)$ .
- p. 364, line 1. Change  $e(D(T))$  to  $e(\text{co}(D(T)))$ .
- p. 370, line 3 of second proof. Change 1.22(d) to 1.23(d).

- p. 377, line 7; p. 378, line 8; page 378, line 10–. Change  $\pi \in B(r, c, t)$  to  $\pi \subseteq B(r, c, t)$ .
- p. 379, line 5–. Insert  $\pi$  after the first “partition”, and change  $\lambda^*$  to  $\pi^*$ .
- p. 379, line 4–. Change “similary” to “similarly”.
- p. 381, line 4–. Change 4.5.6 to 4.5.8.
- p. 383, line 9. Change “ $D(w) = T'$  and  $D(w^{-1}) = T$ ” to “ $D(w) = D(T')$  and  $D(w^{-1}) = D(T)$ ”.
- p. 395, line 10–. Change “Burnside’s theorem” to “Burnside’s lemma”.
- p. 404, line 3–. Littlewood first introduced plethysm in his paper “Polynomial concomitants and invariant matrices,” *J. London Math. Soc.* **11** (1936), 49–55 (page 52).
- p. 416, line 7–. Change  $u_{i_{t+2}}$  to  $u_{j_{t+2}}$ .
- p. 418, line 7. Change “subsequences” to “subsequence”.
- p. 419, line 16. Change “was” to “is”.
- p. 421, line 9–. Insert “a” after “such”.
- p. 421, lines 8– to 7–. Change “second statement of Theorem A1.1.4” to “first assertion of Theorem A1.1.6”.
- p. 422, line 3. Change A1.1.4 to A1.1.6.
- p. 424, line 11. Delete “by”.
- p. 426, line 6–. Change “tableaux in (A1.137)” to “tableau defined by (A1.137)”.
- p. 439, line 7. Delete comma after 156.
- p. 442, Theorem A2.4, line 9. Change “Hence” to “Moreover,”.



- p. 443, line 11. Change

$$\text{char } \varphi = (x_1 \cdots x_n)^{-1} = (x_1 \cdots x_n)^{-1} s_{\emptyset}$$

to

$$\text{char } \varphi = x_1^{-1} + \cdots + x_n^{-1} = (x_1 \cdots x_n)^{-1} s_{1^{n-1}}$$

- p. 444, line 12. Delete “char”.
- p. 444, line 11–. Change “given by (A2.156)” to “generated (as a  $\mathbb{C}$ -algebra) by (A2.156)”.
- p. 447, line 3–. Change  $s_1(x_1^{\lambda_i})$  to  $s_1(x_1^{\lambda_i}, x_2^{\lambda_i}, \dots)$ .
- p. 451, Exercise 7.13(a). This exercise is stated incorrectly. For instance,  $K_{777,6654} = 1$ , contrary to the statement of the exercise. One way to state the correct result is as follows. Let the parts of  $\lambda'$  be given by

$$\begin{aligned} \lambda'_1 = \cdots = \lambda'_{n_1} &> \lambda'_{n_1+1} = \cdots = \lambda'_{n_2} > \lambda'_{n_2+1} = \cdots \\ &> \lambda'_{n_{k-1}+1} = \cdots = \lambda'_{n_k} > 0. \end{aligned}$$

Define  $\lambda^{(j)} = (\lambda'_{n_{j-1}+1}, \dots, \lambda'_{n_j})$  (with  $n_0 = 0$ ), so  $\lambda^{(j)}$  is a partition of rectangular shape. Let  $\mu$  be a partition with  $|\mu| = |\lambda|$ , and let

$$\mu^{(j)} = (\mu_{n_{j-1}+1}, \dots, \mu_{n_j}).$$

Then  $K_{\lambda\mu} = 1$  if and only if  $\lambda \geq \mu$  (dominance order) and

- $|\lambda^{(j)}| = |\mu^{(j)}|$  and  $\lambda^{(j)} \geq \mu^{(j)}$  for all  $j$ .
  - For all  $1 \leq j \leq k$  either  $0 \leq \mu'_{n_{j-1}+1} - \lambda'_{n_{j-1}+1} \leq 1$  or  $0 \leq \lambda'_{n_j} - \mu'_{n_j} \leq 1$ .
- p. 452, line 6. Change “ $k$  times” to “ $n$  times”.
  - p. 452, Exercise 7.16(a), line 5. Change  $c_{i-j} + c_{i+j}$  to  $c_{i-j} - c_{i+j}$ .
  - pp. 452–453, Exercise 7.16(b,e). The formulas for  $y_i(n)$  and  $u_i(n)$  have been extended to  $i \leq 6$  by F. Gascon, *Fonctions de Bessel et combinatoire*, Publ. LACIM **28**, Univ. du Québec à Montréal, 2002 (page 75). In particular,

$$y_6(2n) = 6(2n)! \sum_{k=0}^n \frac{(10n - 13k + 8)C_{k+1}}{(n - k + 2)! (n - k)! (k + 4)! k!},$$

where  $C_{k+1}$  denotes a Catalan number.

- p. 459, Exercise 7.30(c), line 4. Change  $d - 1$  to  $d$ .
- p. 460, Exercise 7.37. For further information on expanding  $a_\delta^2$  in terms of Schur functions, see

<http://www.phys.uni.torun.pl/~bgw/vanex.html>.

- p. 461, Exercise 7.42, line 2. Change  $s_{\bar{\lambda}}(y)$  to  $s_{\tilde{\lambda}}(y)$ .
- p. 466, line 3-. Change  $(\lambda_i - 1)!(\lambda'_i - 1)!$  to  $(\lambda_i - i)!(\lambda'_i - i)!$ .
- p. 467, Exercise 7.55(b). Let  $f(n)$  be the number of  $\lambda \vdash n$  satisfying (7.177). Then

$$(f(1), f(2), \dots, f(30)) = (1, 1, 1, 2, 2, 7, 7, 10, 10, 34, 40, 53, 61, \\ 103, 112, 143, 145, 369, 458, 579, 712, 938, 1127, \\ 1383, 1638, 2308, 2754, 3334, 3925, 5092).$$

The problem of finding a formula for  $f(n)$  was solved by Arvind Ayyer, Amritanshu Prasad, and Steven Spallone, [arXiv:1604.08837](https://arxiv.org/abs/1604.08837).

- p. 467, Exercise 7.59. In order for the bijection  $\lambda \mapsto (\lambda^0, \lambda^1, \dots, \lambda^{p-1})$  given in the solution to part (e) (page 517) to be correct, it is necessary to define a specific indexing of the terms of  $C_\lambda$ . Namely, index a term  $a$  by  $c_i$  if  $i = i_1 - i_0$ , where  $i_1$  is the number of 1's weakly to the left of  $a$ , and  $i_0$  is the number of 0's strictly to the right of  $a$  (so if  $a = 1$ , then this contributes to  $i_1$ ). The sequence becomes  $\dots c_{-2}c_{-1}c_0c_1c_2 \dots$  as before, so it suffices to define the indexing by letting the first 1 be  $c_{i_0-1}$ , where  $i_0$  is the number of 0's following this 1.

*Example.* If  $\lambda = (4, 3, 3, 3, 1)$ , then  $C_\lambda = \dots 0010110001011 \dots$ . The first 1 in this sequence is  $c_{1-4} = c_{-3}$ . On the other hand, if  $\lambda = (3, 3, 3, 2, 2, 1)$ , then  $C_\lambda = \dots 0010100100011 \dots$ . Now the first 1 is  $c_{1-6} = c_{-5}$ .

- p. 468, Exercise 7.59(e), line 3. Change  $Y^k$  to  $Y^p$ .
- p. 469, Exercise 7.61, line 2. Change “0 or 1” to “0 or  $\pm 1$ ”.

- p. 474, Exercise 7.70, line 3. Under the second summation sign insert a space between “in” and  $\mathfrak{S}_n$ .
- p. 485, line 3–. The asymptotic formula for  $a(n)$  should be multiplied by a factor of  $1/\sqrt{3\pi}$ . The factor  $1/\sqrt{\pi}$  was included by Wright and omitted here by mistake. The additional factor  $1/\sqrt{3}$  was omitted by Wright, though his proof makes it clear that it should appear. See L. Mutafchiev and E. Kamenov, [math.CO/0601253](#).
- p. 491, Exercise 7.9, line 1. Insert  $\varepsilon_\lambda$  before  $a_{\lambda\mu}e_\lambda$ .
- p. 492, Exercise 7.11. Change  $\binom{j}{\ell(\mu)-1}$  to  $\binom{\ell(\mu)-1}{j}$  (three times).
- p. 493, Exercise 7.13(a). For another proof, see A. N. Kirillov, *Europ. J. Combinatorics* **21** (2000), 1047–1055, [arXiv:hep-th/9304099](#) (Prop. 2.2).
- p. 494, line 4. Change 169–172 to 175–177.
- p. 494, Figure 7-20. Change the labels  $R_1h6$ ,  $R_1h5$ , and  $R_2h6$  to  $R_1a6$ ,  $R_1a5$ , and  $R_2a5$ , respectively.
- p. 496, equation (7.199). Change  $(m_i(\lambda))^{-1}$  to  $[\prod_i(m_i(\lambda))^{-1}]$ .
- p. 498, Exercise 7.22(h), line 7. Update the Fomin and Greene reference to *Discrete Math.* **193** (1998), 179–200.
- p. 500, displayed tableaux near end of Exercise 7.24. The tableaux  $T_8$  and  $T_9$  are missing the element 8 to the right of 3. Also, the  $\{3, 10\}$  under  $T_9$  should be under  $T_{10}$ .
- p. 502, Exercise 7.27, first displayed equation. Change  $(n)_m$  to  $(n)_{n-m}$ .
- p. 505, Exercise 7.32(a). Stembridge’s more general result appears in “Computational aspects of root systems, Coxeter groups, and Weyl characters,” in *Interactions of Combinatorics and Representation Theory*, MSJ Memoirs **11**, Math. Soc. Japan, Tokyo, 2001, pp. 1–38 (Theorem 7.4).
- p. 506, Exercise 7.37(b), line 2. Change  $s_{\tilde{\chi}}(x)$  to  $s'_{\tilde{\chi}}(x)$ . (The tilde is moved slightly to the left, possibly the most obscure correction appearing in this Supplement.)

- p. 508, Exercise 5.41, line 3 of “*Combinatorial proof.*” Change  $\tilde{T}$  to  $T'$ .
- p. 514, Exercise 7.47(m), lines 1–3. Update the reference to R. Stanley, *Discrete Math.* **193** (1998), 267–286.
- p. 514, Exercise 7.47(m). The conjecture of Hamidoune was proved (without using symmetric functions) by M. Chudnovsky and P. D. Seymour, *J. Comb. Theory, Ser. B* **97** (2007), 350–357.
- p. 514, Exercise 7.48(b), lines 2–4. Update the reference to R. Simion and R. Stanley, *Discrete Math.* **204** (1999), 369–396.
- p. 514, line 1–, and p. 515, line 1. “*Ibid.*” refers to the reference in the item above, not to the previous reference in the book.
- p. 515, line 3. “the reference” refers to the reference two items above, viz., R. Stanley, *Electron. J. Combinatorics* **3**, R6 (1996), 22 pp.
- p. 515, Exercise 7.49. Update this reference to C. Lenart, *J. Algebraic Combin.* **11** (2000), 69–78.
- p. 516, line 8. Change  $(\lambda_i - 1)!(\lambda'_i - 1)!$  to  $(\lambda_i - i)!(\lambda'_i - i)!$ .
- p. 516, Exercise 7.54. The following elegant solution is due to Katerina Kalampogia-Evangelinou. Expand  $s_\lambda$  in terms of power sums and set  $x_i = q^{i-1}$  (principal specialization). If  $\mu$  has no even part, then  $p_\mu(1, q, q^2, \dots)$  has no pole at  $q = -1$ . If  $\lambda$  has an even hook length, then by Corollary 7.21.3  $s_\lambda(1, q, q^2, \dots)$  has a pole at  $q = -1$ , and the proof follows.
- p. 517, Exercise 7.59(e), line 3. Change  $Y^k$  to  $Y^p$ .
- p. 517, Exercise 7.59(e), line 10. Change  $Y_\emptyset$  to  $Y_{p,\emptyset}$ , and change  $Y^k$  to  $Y^p$ .
- p. 518, Exercise 7.59(h), line 1. Change  $Y_\emptyset$  to  $Y_{p,\emptyset}$ , and change  $Y^k$  to  $Y^p$ .
- p. 518, Exercise 7.59(h), line 2. Change  $Y^k$  to  $Y^p$  (three times).
- p. 518, Exercise 7.59(h), line 3. Change  $Y^k$  to  $Y^p$ .

- p. 520, line 3-. Change  $\sum_{n \geq 0} h_{2n+1} t^{2n+1}$  to  $\sum_{n \geq 0} (-1)^n h_{2n+1} t^{2n+1}$
- p. 535, lines 7–10. Replace the sentence “No proof ... are known.” with “A bijective proof of the unimodality of  $s_\lambda(1, q, \dots, q^n)$  was given by A. N. Kirillov, *C. R. Acad. Sci. Paris*, Sér. I **315** (1992), 497–501.”
- p. 537, Exercise 7.78(f), line 6. Change  $s_\mu(x)$  to  $s_\mu(y)$  and  $s_\nu(x)$  to  $s_\nu(z)$ .
- p. 544, part (e), lines 1–2. Change “the dual Jacobi-Trudi identity (Corollary 7.16.2)” to “iterating the dual Pieri identity (page 340)”.
- p. 550, Exercise 7.100(c), line 3. Insert } after  $\leq a$ .
- p. 576, line 7. Change work to word.
- p. 580. Change “Valquelin, D.” to “Vauquelin, B.”.