

How to Win the Lottery

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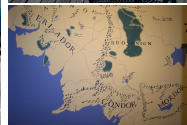
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The Disclaimer

Most Lotteries Will Not Make You Rich

The Place: Random Hall



The Time: IAP 2005



The Players: Reid, James, Yuran, et al



The Game: Cash Winfall

- Pick **6** numbers, each from **1** to **46**
- If you match all six, you win!
- If you match five, four, three or two you win too, but less.
- Key: when the jackpot hits \$2, 000, 000 **rolldown**.

The Prizes: Millions

	Normal Prize	Rolldown Prize	Rolldown Percentage
Match 2	Free Play	Free Play	0%
Match 3	\$5	\$26.85	27%
Match 4	\$150	\$807.52	47%
Match 5	\$4,000	\$22,096	26%

The Odds: Good Enough

Assume nobody wins the jackpot.

There are a total of $\binom{46}{6} = 9,366,819$ possible tickets.

	Matches	Odds	Normal EV	Rolldown EV
Match 2	$\binom{6}{2}\binom{40}{4} = 1,370,850$	1/7	\$0.29	\$0.29
Match 3	$\binom{6}{3}\binom{40}{3} = 197,600$	1/47	\$0.11	\$0.57
Match 4	$\binom{6}{4}\binom{40}{2} = 11,700$	1/800	\$0.19	\$1.01
Match 5	$\binom{6}{5}\binom{40}{1} = 240$	1/39k	\$0.10	\$0.57
Match 6	$\binom{6}{6}\binom{40}{0} = 1$	1/9.4M	–	–
Total			\$0.69	\$2.43

The Catch: Jackpot!

We lose if someone else wins the jackpot. Say 1,250,000 tickets bought, \$2,400,000 jackpot afterward.

- The chances of someone winning are about 1 in 7.5.
- In this case, we get \$0.69 back per ticket bought and lose 2/3 of our investment.
- Unless it was us.
- The real EV is
$$(0.133)(\$0.69) + (0.867)(\$2.43) + (1/9.4M)(\$2.4M) = \$2.46.$$

This happened once, on July 10, 2008.

The Decision: Go/No Go

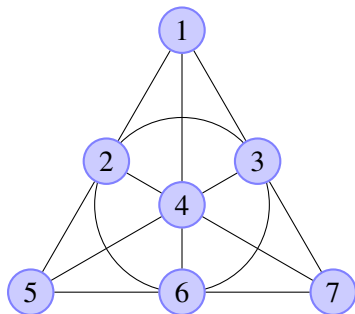
- Very different percentile returns from a normal lottery.
- James and Yuran started charging a fee to run the pool (it was a lot of work!)
- Is it really worth it: utility functions.

The Numbers: Toy Demo

Let's play a game.

- The lottery picks 3 numbers from 1 to 7
- Each ticket wins \$6 if you match all, or \$2 if you match 2.
- If you win at least \$6, you get to play in the next round.
- You get to select seven triples to play (out of the 35 possibilities). What do you do?

The Numbers: Projective Plane Solution



Get the following triples: 125, 137, 146, 236, 247, 345, 567.

Note: the following would be a bad choice: 123, 124, 125, 134, 135, 234, 235

The Numbers: Combinatorial Designs

Definition

A set S of q -subsets of an n -set X is a *design with parameters* (n, q, r, λ) if every r -subset of X belongs to exactly λ elements of S .

Projective plane is a $(7, 3, 2, 1)$ -design. In order for a design to exist, one must have the following divisibility conditions:

$$\binom{q-i}{r-i} \text{ divides } \lambda \binom{n-i}{r-i} \text{ for } 0 \leq i \leq r-1. \quad (1)$$

Theorem (Keevash, 2014)

If (n, q, r, λ) satisfies (1) then a design exists.

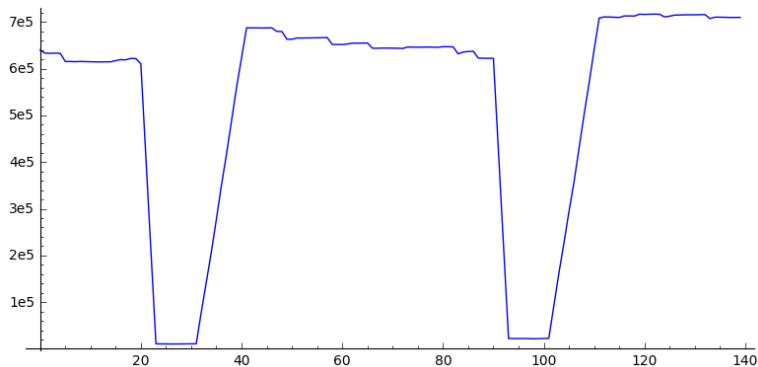
In particular, $(46, 6, 4, \lambda)$ designs exist when $3 \mid \lambda$, and $|S| = \lambda \frac{\binom{46}{4}}{\binom{6}{4}} = 10879\lambda$.

Remark

Actually, found an approximate solution by greedily minimizing variance.

The Unexpected: Frozen!

The following is a guess at what the balance in James' bank account might look like, as a function of time (in days).



The Scheme: Early Rolldown

- The lottery posts projections for how big the jackpot will be.
- In one marginal case, they projected a rollover which didn't happen.
- If the previous jackpot was under \$1,600,000, the lottery tended not to project a rolldown.
- On August 16, 2010, Random Strategies induced an early rolldown by buying 700,000 tickets.
- Made \$700,000 profit on \$1,400,000 investment.

The End: Newspapers

- At the end of July, 2011, the Boston Globe published an article describing Cash Winfall and the betting groups.
- After public outcry, the Lottery restricted the ability of stores to sell huge numbers of tickets.
- The game was discontinued in January, 2012.

The Ethics: Should We Have Lotteries

- The state took a 40% cut of every ticket sold.
- Massachusetts knew about the betting groups all along.
- States tend to use lottery funds for popular programs (e.g. education or environment). But legislators account for these funds when setting budgets.
- Video lotteries a lot more addictive ([Last Week Tonight episode](#) by John Oliver).

The Alternatives: Other States

- Cash Winfall evolved out of a similar game in Michigan.
- Some lotteries have an effective rolldown when they end a game
 - ▶ Pennsylvania: Random Strategies made \$1,000,000 on \$600,000 played.
- Sometimes lotteries go a long time without a winner (but beware taxes and splitting!)
 - ▶ Indiana: jackpot reached \$54.5 million, with no winners from October 21, 2006 to November 7, 2007.
 - ▶ Also pick 6 of 46, so chances of winning are one in 9.37 million, and buying all of the tickets costs \$18.7 million.

The Warning: Think before you Play Much!

Very, very few lotteries are worth playing as an investment!

Learn More

- *Jerry and Marge Go Large* (2022 movie)
- Massachusetts Office of the Inspector General Report (25 pages).
- Jordan Ellenberg, *How Not to Be Wrong*.
- Boston Globe, *High-rollers profit from Cash Winfall*.