How to Win the Lottery

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The Disclaimer

Most Lotteries Will Not Make You Rich
The Place: Random Hall
The Players: Reid, James, Yuran, et al
The Game: Cash Winfall

- Pick 6 numbers, each from 1 to 46
- If you match all six, you win!
- If you match five, four, three or two you win too, but less.
- Key: when the jackpot hits $2,000,000 rolldown.
## The Prizes: Millions

<table>
<thead>
<tr>
<th></th>
<th>Normal Prize</th>
<th>Rolldown Prize</th>
<th>Rolldown Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match 2</td>
<td>Free Play</td>
<td>Free Play</td>
<td>0%</td>
</tr>
<tr>
<td>Match 3</td>
<td>$5</td>
<td>$26.85</td>
<td>27%</td>
</tr>
<tr>
<td>Match 4</td>
<td>$150</td>
<td>$807.52</td>
<td>47%</td>
</tr>
<tr>
<td>Match 5</td>
<td>$4,000</td>
<td>$22,096</td>
<td>26%</td>
</tr>
</tbody>
</table>
The Odds: Good Enough

Assume nobody wins the jackpot.
There are a total of $\binom{46}{6} = 9,366,819$ possible tickets.

<table>
<thead>
<tr>
<th>Matches</th>
<th>Odds</th>
<th>Normal EV</th>
<th>Rolldown EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match 2</td>
<td>$\binom{6}{2}\binom{40}{4} = 1,370,850$</td>
<td>1/7</td>
<td>$0.29$</td>
</tr>
<tr>
<td>Match 3</td>
<td>$\binom{6}{3}\binom{40}{3} = 197,600$</td>
<td>1/47</td>
<td>$0.11$</td>
</tr>
<tr>
<td>Match 4</td>
<td>$\binom{6}{4}\binom{40}{2} = 11,700$</td>
<td>1/800</td>
<td>$0.19$</td>
</tr>
<tr>
<td>Match 5</td>
<td>$\binom{6}{5}\binom{40}{1} = 240$</td>
<td>1/39k</td>
<td>$0.10$</td>
</tr>
<tr>
<td>Match 6</td>
<td>$\binom{6}{6}\binom{40}{0} = 1$</td>
<td>1/9.4M</td>
<td>–</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>$0.69$</td>
</tr>
</tbody>
</table>
The Catch: Jackpot!

We lose if someone else wins the jackpot. Say 1, 250, 000 tickets bought, $2, 400, 000 jackpot afterward.

- The chances of someone winning are about 1 in 7.5.
- In this case, we get $0.69 back per ticket bought and lose 2/3 of our investment.
- Unless it was us.
- The real EV is

$\begin{align*}
(0.133)(0.69) + (0.867)(2.43) + (1/9.4M)(2.4M) &= 2.46.
\end{align*}$

This happened once, on July 10, 2008.
The Decision: Go/No Go

- Very different percentile returns from a normal lottery.
- James and Yuran started charging a fee to run the pool (it was a lot of work!)
- Is it really worth it: utility functions.
Let’s play a game.

- The lottery picks 3 numbers from 1 to 7
- Each ticket wins $6 if you match all, or $2 if you match 2.
- If you win at least $6, you get to play in the next round.
- You get to select seven triples to play (out of the 35 possibilities). What do you do?
Get the following triples: 125, 137, 146, 236, 247, 345, 567.
Note: the following would be a bad choice: 123, 124, 125, 134, 135, 234, 235
The Numbers: Combinatorial Designs

Definition
A set $S$ of $q$-subsets of an $n$-set $X$ is a design with parameters $(n, q, r, \lambda)$ if every $r$-subset of $X$ belongs to exactly $\lambda$ elements of $S$.

Projective plane is a $(7, 3, 2, 1)$-design. In order for a design to exist, one must have the following divisibility conditions:

\[
\binom{q-i}{r-i} \text{ divides } \lambda \binom{n-i}{r-i} \text{ for } 0 \leq i \leq r - 1.
\]  

Theorem (Keevash, 2014)
If $(n, q, r, \lambda)$ satisfies (1) then a design exists.

In particular, $(46, 6, 4, \lambda)$ designs exist when $3 \mid \lambda$, and $|S| = \lambda \frac{46}{4} \frac{6}{4} = 10879 \lambda$.

Remark
Actually, found an approximate solution by greedily minimizing variance.
The Unexpected: Frozen!

The following is a guess at what the balance in James’ bank account might look like, as a function of time (in days).
The lottery posts projections for how big the jackpot will be.

In one marginal case, they projected a rollover which didn’t happen.

If the previous jackpot was under $1,600,000, the lottery tended not to project a rolldown.

On August 16, 2010, Random Strategies induced an early rolldown by buying 700,000 tickets.

Made $700,000 profit on $1,400,000 investment.
At the end of July, 2011, the Boston Globe published an article describing Cash Winfall and the betting groups.

After public outcry, the Lottery restricted the ability of stores to sell huge numbers of tickets.

The game was discontinued in January, 2012.
The Ethics: Should We Have Lotteries

- The state took a 40% cut of every ticket sold.
- Massachusetts knew about the betting groups all along.
- States tend to use lottery funds for popular programs (e.g. education or environment). But legislators account for these funds when setting budgets.
- Video lotteries a lot more addictive (Last Week Tonight episode by John Oliver).
The Alternatives: Other States

- Cash Winfall evolved out of a similar game in Michigan.
- Some lotteries have an effective rolldown when they end a game
  - Pennsylvania: Random Strategies made $1,000,000 on $600,000 played.
- Sometimes lotteries go a long time without a winner (but beware taxes and splitting!)
  - Indiana: jackpot reached $54.5 million, with no winners from October 21, 2006 to November 7, 2007.
  - Also pick 6 of 46, so chances of winning are one in 9.37 million, and buying all of the tickets costs $18.7 million.
The Warning: Think before you Play Much!

Very, very few lotteries are worth playing as an investment!
Learn More

- *Jerry and Marge Go Large* (2022 movie)
- Jordan Ellenberg, *How Not to Be Wrong*.
- Boston Globe, *High-rollers profit from Cash Winfall*. 