Homework 24 Solutions

Problems

1. **Compute the following:**

   (a) $\phi(29)$. **(Here $\phi(n)$ is Euler's phi function, as discussed in lecture.)**

   (b) $\phi(116)$.

   (c) $\phi(6615)$.

   $\phi(29) = \boxed{28}$ since 29 is prime.

   $\phi(116) = (1/2)(28/29)116 = \boxed{56}$.

   $6615 = 3^3 \times 5 \times 7^2$ so $\phi(6615) = (2/3)(4/5)(6/7)6615 = \boxed{3024}$

2. $3^{28} = 1 \pmod{29}$ **by Fermat's Little Theorem (since 29 is prime).**

   Compute $3^{56} \pmod{116}$ using the Chinese Remainder Theorem.

   Since $116 = 29 \cdot 4$, it suffices to figure out $3^{56} \pmod{29}$ and $3^{56} \pmod 4$.

   Since $3^{28} \equiv 1 \pmod{29}$ we also have $3^{56} \equiv 1 \pmod{29}$.

   Since $3^2 \equiv 1 \pmod 4$, we have $3^{56} \equiv 1 \pmod 4$.

   By the Chinese Remainder Theorem, $3^{56} \equiv \boxed{1} \pmod{116}$.

   Note that $\phi(116) = 56$. We now know that this is an example of Euler's Theorem at work.