Homework 13 Solutions

Problems

1. **Factor the following into prime numbers:**

    (a) $\binom{20}{6}$; (***Hint:*** **You do not need to compute the actual number.**)

    (b) **6006;**

    (c) **2006.**

$$
\begin{aligned}
\binom{20}{6} &= \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
&= \frac{5 \times 4 \times 19 \times 6 \times 3 \times 17 \times 2 \times 8 \times 15}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
&= 19 \times 17 \times 8 \times 15 \\
&= 2^3 \times 3 \times 5 \times 17 \times 19
\end{aligned}
$$

$$
\begin{aligned}
6006 &= 6 \times 1001 \\
&= 2 \times 3 \times 7 \times 11 \times 13
\end{aligned}
$$

$$
\begin{aligned}
2006 &= 2 \times 1003 \\
&= 2 \times 17 \times 59
\end{aligned}
$$

2. (a) **Is $\binom{19}{7}$ divisible by 13?**

    (b) **Is $\binom{19}{7}$ divisible by 19?**

    (c) **Is $\binom{19}{7}$ divisible by 23?**

$$
\binom{19}{7} = \frac{19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{7 \times 6 \times 5 \times 4 \times 3 \times 2}
$$

First note that 13, 19, and 23 are all prime, so in order to check if they divide a binomial coefficient, we need only check to see how many times they divide the numerator and denominator.

13 divides the top of the fraction, but not the bottom, hence 13 does divide $\binom{19}{7}$.

19 divides the top of the fraction, but not the bottom, hence 19 does divide $\binom{19}{7}$.

23 does not divide the top of the fraction (or the bottom), hence 23 does not divide $\binom{19}{7}$.