Quantitative Reasoning 28:
The Magic of Numbers

Homework 21

Assigned on Friday November 14th
Due at 12 noon Monday November 17th

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 18

Problems:

Please explain your reasoning and show your work.
Here is the table of powers modulo 13 that you computed on HW 20:

x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
1	1	1	1	1	1	1	1	1	1	1	1
2	4	8	3	6	12	11	9	5	10	7	1
3	9	1	3	9	1	3	9	1	3	9	1
4	3	12	9	10	1	4	3	12	9	10	1
5	12	8	1	5	12	8	1	5	12	8	1
6	10	8	9	2	12	7	3	5	4	11	1
7	10	5	9	11	12	6	3	8	4	2	1
8	12	5	1	8	12	5	1	8	12	5	1
9	3	1	9	3	1	9	3	1	9	3	1
10	9	12	3	4	1	10	9	12	3	4	1
11	4	5	3	7	12	2	9	8	10	6	1
12	1	12	1	12	1	12	1	12	1	12	1

1. Use the table of powers   (mod 13) to compute the following:

   (a) What is the 5th root of 4 (mod 13)?

   (b) What is the 11th root of 9 (mod 13)?

   (c) What is the 7th root of 3 (mod 13)?

2. (a) How many 8th roots does 9 have   (mod 13)? How many 8th roots does 3 have   (mod 13)? How many 8th roots does 7 have   (mod 13)?

(b) How many 9th roots does 8 have (mod 13)? How many 9th roots does 6 have (mod 13)? How many 9th roots does 5 have (mod 13)?

(c) What is the greatest common divisor of 8 and $13 - 1$?

(d) What is the greatest common divisor of 9 and $13 - 1$?

3. Suppose I tell you that 39847418273263 is prime, and that

$$5441662622048^{19} \equiv 2673482^{19} \pmod{39847418273263}.$$

Note that $2^{19} = 524288$. How many 19th roots of 524288 are there (mod 39847418273263), and why?

4. The goal of this exercise is to compute 5^{82} (mod 103) using the method outlined in Section 18.2 of the textbook.

(a) Write 82 as a sum of powers of 2. What is the largest power of 2 appearing?

(b) Compute 5^2 (mod 103). Compute 5^4 (mod 103). Compute 5^8 (mod 103). Keep going until you've computed 5 raised to the largest power of 2 appearing in part (a).

(c) Use part (a) to write 5^{82} as a product of the numbers you computed in part (b). Multiply these out (mod 103) in order to find the final answer.