Quantitative Reasoning 28:
The Magic of Numbers

Homework 20

Assigned on Friday November 7th
Due at 12 noon Monday November 10th

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 18

Problems:

Please explain your reasoning and show your work.

1. (a) What does Fermat's Theorem say about powers (mod 53)?
   (b) Compute 3^{109} (mod 53).
   (c) Compute 2^{270} (mod 53).

2. (a) What is the last digit of 3^{991}?
   (b) Compute 3^{991} (mod 11).
   (c) Compute 26^{991} (mod 13).

3. (a) Create a power table for arithmetic (mod 13). This will be a table whose rows correspond to numbers in arithmetic (mod 13) (that is, the numbers $\{0, 1, 2, \ldots, 12\}$), and whose entries are their various powers. Compute the powers from the 1st up to the 13th power for each number. (Remember, for example, that $10 = -3$ (mod 13) and you can use this to avoid doing the computations for 10 once you've done them for 3.)
   (b) Compute 2^{742} (mod 13).