Quantitative Reasoning 28:  
The Magic of Numbers

Homework 20

Assigned on Friday November 7th  
Due at 12 noon Monday November 10th

Please submit problem sets at the end of the relevant lecture, or leave in the  
box labeled QR28 outside the Math Department’s main office, on the third floor  
of the Science Center (Room 325).

Reading:  
Gross-Harris, Chapter 18

Problems:

Please explain your reasoning and show your work.

1. (a) What does Fermat’s Theorem say about powers $\pmod{53}$?  
   (b) Compute $3^{109} \pmod{53}$.  
   (c) Compute $2^{270} \pmod{53}$.  

2. (a) What is the last digit of $3^{991}$?  
   (b) Compute $3^{991} \pmod{11}$.  
   (c) Compute $26^{991} \pmod{13}$.

3. (a) Create a power table for arithmetic $\pmod{13}$. This will be a table  whose rows correspond to numbers in arithmetic $\pmod{13}$ (that is,  the numbers $\{0,1,2,\ldots,12\}$), and whose entries are their various  powers. Compute the powers from the 1st up to the 13th power for each  number. (Remember, for example, that $10 = -3 \pmod{13}$ and  you can use this to avoid doing the computations for 10 once you’ve done them for 3.)  
   (b) Compute $2^{742} \pmod{13}$.