Quantitative Reasoning 28:
The Magic of Numbers

Homework 17

Assigned on Friday, October 31
Due at 12 noon, Monday, November 3

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 16 and 17

Problems:

Please explain your reasoning and show your work.

1. Do the following computations in the given modulus.

   (a) $6 - 4 \pmod 7$.
   (b) $80 + 21 \pmod{101}$.
   (c) $3 - 12 \pmod{15}$.
   (d) $456 \times 450 \pmod{457}$.

2. Do the following computations in the given modulus.

   (a) $457 \times 458 \pmod{459}$.
   (b) $13 \times 44 \pmod 5$.
   (c) $13 \times 44 \pmod{15}$.

   (Can you explain what is going on here?)

3. This problem concerns a divisibility rule for 4, ie a way of telling if a number is divisible by 4 easily.

(a) Show why the following divisibility rule for 4 works:

Add the last digit to twice the second to last digit. If the sum is a multiple of four, then the number is a multiple of 4.

For example, 16 is a multiple of 4 since $6 + 2 \cdot 1 = 8$ is a multiple of 4.

[Hint: We can write 538 as $5 \cdot 10^2 + 3 \cdot 10 + 8$. What does considering this expression modulo 4 tell us?]

(b) Is 2736 divisible by 4? Why or why not?

(c) Is 29384710293847019283470192 8374 divisible by 4? Why or why not?