Practice Midterm 1

You may want to take this with the book closed and with no calculator. However, this is longer than the real midterm will be.

1. (a) **How many numbers are there between** 1776 **and** 2005 **(inclusive) which are divisible by** 2 **but not by** 4**?**
   (b) **How many numbers in the previous problem are not divisible by** 3**?**

2. (a) **How many numbers are there between 60 and 150, inclusive?**
   (b) **Of the numbers in (a), how many are divisible by 2? How many are divisible by 3?**
   (c) **Of the numbers in (a), how many are divisible by 2 but not by 3?**
   (d) **Of the numbers in (a), how many are divisible by 2 *or* by 3 (or both)? (Examples of such numbers are 96, 99, 100.)**

3. **The Greek alphabet has** 24 **letters,** 7 **of which** (A, E, H, I, O, Υ, Ω) **are vowels.**
   (a) **How many possible five-letter Greek words are there? ("Mathematical words", that is: as usual we do not ask that the words make any sense — it's all Greek to us in any case.)**
   (b) **Of the five-letter Greek words, how many contain exactly two vowels?**
   (c) **A *palindrome* is a word, such as** SEES **or** DEIFIED **or** E$\Omega\Phi\Phi\Omega$E**, that reads the same forward and back. Of the five-letter Greek words with exactly two vowels, how many are palindromes?

4. (a) **Suppose that a license plate has seven spots, and that valid characters for these slots are the 26 letters (A through Z) and the 10 digits (0 through 9). How many possible license plates are there?**
   (b) **Now suppose that you want your license plate to contain exactly 3 letters. How many possible plates are there?**
   (c) **Now suppose that, in addition to having exactly 3 letters, you also refuse to allow these letters to be all in a row. How many possible plates are there meeting these conditions?**

5. (a) **You are dealt, at random, 4 cards from a standard 52 card deck. What is the probability that the denominations are all consecutive (that is, you have the 4-card version of a straight)? Here the Ace can be either high or low, so that** $\{A, 2, 3, 4\}$ **and** $\{J, Q, K, A\}$ **both count.**
   (b) **What is the probability that you have 2 pair, that is, that you have two cards of one denomination and two cards of a different denomination?**

6. **You are playing a slightly unorthodox version of Scrabble where you are given ten letters (rather than the usual seven). Amazingly enough, you drew (randomly) from the pile precisely these ten letters: ACCELERATE, but probably not in exactly that order. Given that you drew this set of ten Scrabble tiles, what is the probability that you did, in fact, draw them from the pile spelling the word ACCELERATE?**

7. **Yahtzee is played with** 5 **dice, each numbered in the usual way from** 1 **to** 6 **and equally likely to land on each face.**
   (a) **What is the probability of the total on the** 5 **dice being <u>at least</u>** 29**?**

(b) **Suppose now that one of the dice is marked dishonestly, with the 1-face replaced by a second 6 (so the faces are $2, 3, 4, 5, 6, 6$ instead of the honest $1, 2, 3, 4, 5, 6$). What is the probability of scoring at least 29 with this die and four honest dice?**

8. **At parties, local mathemagician Polly Nomial often produces an ordinary pack of cards and invites a stranger to choose three at random. Polly will then claim that at least one of the chosen cards is an Ace. The thing is: there's no trick involved! What's the probability that Polly's claim is correct (i.e., that at least one of the three is an Ace)?**

9. (a) **Compute $\binom{16}{3}$ as a whole number.**

   (b) **Compute $\binom{17}{14}$ as a whole number.**

   (c) **Yes or no, and why: Is $\binom{106}{102}$ bigger than $\binom{105}{4} + \binom{105}{3}$?**

10. **Rewrite the following in a simpler form, using the binomial theorem (NB: the binomial theorem will not be on the exam. But we wanted to give you a problem to practice using it, for your own edification).**

$$\binom{6}{0} + 2 \cdot \binom{6}{1} + 4 \cdot \binom{6}{2} + 8 \cdot \binom{6}{3} + 16 \cdot \binom{6}{4} + 32 \cdot \binom{6}{5} + 64 \cdot \binom{6}{6}$$