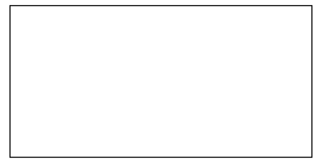
Math 430 Midterm Exam

Name	
Problem 1 (18 points)	
Problem 2 (26 points)	
Problem 3 (16 points)	
Problem 4 (8 points)	
Problem 5 (8 points)	
Problem 6 (Bonus)	
Total (76 points)	

1. Consider the symmetry group G of a rectangle with side lengths 1 and 2.



   (a) (4 points) List the elements of G.

   (b) (4 points) Show that G is abelian.

   (c) (5 points) List the subgroups of G.

   (d) (5 points) Do there exist nontrivial subgroups H and K so that G is an internal direct product of H and K? Explain.

2. Let $\sigma = (123)(4567) \in S_7$.

   (a) (5 points) What is the order of σ? Explain.

   (b) (5 points) Find σ^{-1}.

   (c) (5 points) Is σ even or odd? Why?

   (d) (5 points) Give an isomorphism between the subgroup $\langle\sigma\rangle$ generated by σ and $\mathbb{Z}_n$ for some n.

   (e) (6 points) Find all $\tau \in S_7$ that also generate $\langle\sigma\rangle$, i.e. all τ with $\langle\tau\rangle = \langle\sigma\rangle$.

3. Suppose H and K are normal subgroups of a group G.

   (a) (8 points) Prove that $H \cap K$ is a subgroup of G.

   (b) (8 points) Prove that $H \cap K$ is a normal subgroup of G.

4. (8 points) Consider the subgroup $H = D_5$ of $G = S_5$. How many cosets does H have in G? Justify your answer.

5. (8 points) Suppose G is a group and $g, h \in G$. Prove that the order of hgh^{-1} is the same as the order of g.

6. (Bonus) Prove that there is no cyclic group G that has 90 different generators (i.e. $G = \langle g \rangle$ for 90 different $g \in G$).