Math 400 - Practice Final: Part 1

1. Consider the line L with equation $2x + 3y = 12$.
   (a) What is the slope of L?
   (b) Find an equation for the line M through $(6, 4)$ that is perpendicular to L.
   (c) Find the coordinates of the point P where M intersects L.
   (d) Find the distance between P and $(6, 4)$.

2. Consider the system

$$\begin{aligned} 3y - 3z &= 3 \\ x - y + 2z &= 5 \\ 2x - 3y + 5z &= 9 \end{aligned}$$

   (a) Write down the augmented matrix corresponding to this system.
   (b) Find the row-reduced echelon form of this matrix.
   (c) Does this system have a unique solution, infinitely many solutions or no solutions? Why? If there are solutions, write down a general form for the solution.

3. Solve the system

$$\begin{aligned} 3x - 5y &= 2 \\ -x + 3y &= 4 \end{aligned}$$

   using matrix inverses.

4. Construct the truth table for the compound proposition $(p \to q) \leftrightarrow (p \wedge q)$.

5. For each of the following propositions, use *exactly one* of the laws of logic to transform it into an equivalent proposition.
   (a) $p \wedge (q \vee r)$.
   (b) $\sim (p \wedge q)$.
   (c) $p \wedge p$.

6. Determine whether the following argument is valid:

   If Al goes to the gym, then he is an athlete.
   If Brenda does not go to the gym, then Cindy lifts weights.
   Cindy does not lift weights

   Therefore, Al does not go to the gym, and Brenda does not go to the gym.

7. What is the effective rate of return on a savings account that pays 5% nominal interest, compounded monthly? You do not need to simplify your answer.

8. If you deposit \$100 per month for one year, earning 12% interest compounded monthly, how much money will you have at the end of the year? (You make a total of 12 deposits, and the first earns interest 12 times)