Math 400 - Practice Exam 2 Solutions

1. We compute the effective rates for the two accounts.

   (a) For the first account, $r = 0.08$ and $m = 12$. So $r_{\text{eff}} = (1 + \frac{0.08}{12})^{12}$.

   (b) For the second account, $r = 0.07$ and $m = 365$. So $r_{\text{eff}} = (1 + \frac{0.07}{365})^{365}$.

   Whichever effective rate is larger will produce the larger return. In this case, the 8% nominal rate is better (but you wouldn't need to compute this on an exam).

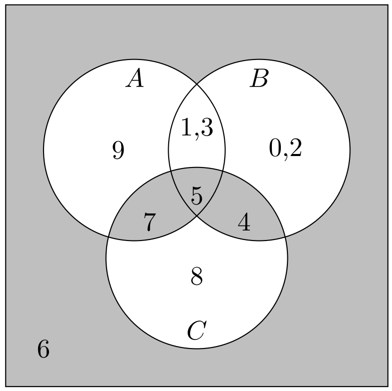
2. (a) If it is an arithmetic sequence, it must have common difference 2 because 5 and 7 differ by 2. Filling in the rest of the terms gives $3, 5, 7, 9, 11$. Since this agrees with the other specified term (11), this is an arithmetic progression. If it were a geometric progression, it would have to have common ratio $\frac{7}{5}$, which would make the last term $5 \cdot \left(\frac{7}{5}\right)^3 \neq 11$.

   The sum of the first n terms is $a_1 + \frac{n(n-1)}{2}d$, so the sum of the first ten terms is $3 + \frac{10 \cdot 9}{2} \cdot 2 = 93$.

   (b) If it were an arithmetic progression, it would have to have common difference 2, making the first term 4. So it it not arithmetic. If it were a geometric progression, it would have common ratio $\frac{6}{5}$, and then the third term would be $\left(\frac{6}{5}\right)^3 \neq 10$. So it is not geometric.

   (c) If it were arithmetic, it would have to have common difference $\frac{6-3}{2}$, making the last term 9, not 12. So it is not arithmetic. If it were geometric with common ratio r, then r would satisfy $3r^2 = 6$, so $r = \sqrt{2}$ (since all numbers are positive). The first five terms would then be $3, 3\sqrt{2}, 6, 6\sqrt{2}, 12$, lining up with what is given.

   The sum of the first n terms is $a_1 \frac{r^n - 1}{r - 1}$, so the sum of the first ten terms is $3 \frac{\sqrt{2}^{10} - 1}{\sqrt{2} - 1} = 93(\sqrt{2} + 1)$.

3. 

Since C has 4 elements, it has $2^4 = 16$ subsets.

4. (a) For a straight flush, there are 4 choices for the suit, and 10 choices for the starting card (from A to 10). Thus the probability of drawing a straight flush is

$$\frac{40}{\binom{52}{5}} \approx 0.0000154$$

(you don't need to give this numerical value).

(b) For a straight, there are 4^5 choices for suit and 10 choices for the starting card. For a flush, there are 4 choices for the suit and $\binom{13}{5}$ for the ranks. We've double counted the straight flushes, which are precisely the intersection of the two. So the probability of drawing either a straight or a flush is

$$\frac{4^5 \cdot 10 + 4 \cdot \binom{13}{5} - 40}{\binom{52}{5}} \approx 0.00591$$

(you don't need to give this numerical value).

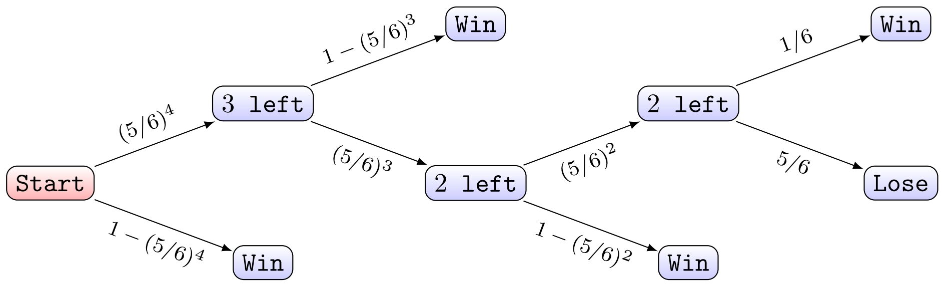
5. (a) The sample space is the sequence of four numbers that appear on the top of the dice (there are a total of 6^4 elements in the sample space).

(b) There are 6 options for the pair, and then 6 options for the third die and 6 for the fourth. So the probability of E is $\frac{6^3}{6^4} = \frac{1}{6}$.

(c) There are 6 options for the value of the first pair and 6 options for the value of the second pair. We then need to multiply by the number of ways we can choose the location of these pairs among the four dice. There are 3 of these, since there are three dice that can match the first die, then the other two dice are forced to match (you can also list them: AABB, ABAB and ABBA). So the overall probability is $\frac{3 \cdot 6^2}{6^4} = \frac{1}{12}$.

(d) The probability that both E and F occur is $\frac{1}{36}$, since we no longer have the flexibility to choose which dice are paired with each other (we must have the pattern AABB). Since $\frac{1}{36} \neq \frac{1}{6} \cdot \frac{1}{12}$, the two events are not independent.

6. (a)



(b) The probability of not rolling any 6s is $(5/6)^4$, since the probability on each die is 5/6 and they are independent of each other. So the probability of rolling at least one 6 is $1-(5/6)^4$.

(c) For each of the four possible Win branches, the probability of reaching that branch is the product of the probabilities along the edges to that branch. The total probability of winning is then the sum of these:

$$\begin{aligned}\big(1-(5/6)^4\big)+\big((5/6)^4(1-(5/6)^3)\big)+\big((5/6)^7(1-(5/6)^2)\big)+\big((5/6)^9(1-5/6)\big)\\ = 1-(5/6)^{10} \approx 0.84.\end{aligned}$$

Note that we can also compute this probability directly without referring to the tree diagram, since the complement is the probability of rolling ten dice in a row, none of which are a 6.

(d) This is the same kind of probability as the previous part, but starting with two dice instead of four. So,

$$\big(1-(5/6)^2\big)+\big((5/6)^2(1-(5/6))\big) = 1-(5/6)^3 \approx 0.42$$