

Math 400 - Practice Exam 2

1. Consider two bank accounts: the first pays an 8% nominal interest rate, compounded monthly; the second pays a 7% nominal interest rate, compounded daily. Explain how to determine which will provide a larger return over the course of a year.
2. For each of the following sequences of positive numbers, determine whether it is an arithmetic progression, geometric progression, both or neither. Then fill in the missing terms, and give an expression for the sum of the first ten terms.
 - (a) $_, 5, 7, _, 11, \dots$
 - (b) $1, _, _, 10, 12, \dots$
 - (c) $3, _, 6, _, 12, \dots$

3. Let

$$S = \{\text{integers } n : 0 \leq n \leq 9\},$$

$$A = \{n \in S : n \text{ is odd}\},$$

$$B = \{n \in S : n < 6\},$$

$$C = \{4, 5, 7, 8\}.$$

- (a) Draw and label a Venn diagram (with universe S), putting the elements of S into the appropriate regions.
 - (b) On your diagram, shade in the region corresponding to $(A \cup B) \cap C \cup (A \cup B \cup C)^c$.
 - (c) How many subsets does C have?
4. Recall that a hand of five cards drawn from a standard deck of 52 cards is a *straight* if it consists of five cards in sequence (ranging from $A, 2, 3, 4, 5$ to $10, J, Q, K, A$ with no restriction on suit). A *flush* consists of any five cards of the same suit, and a *straight flush* is simultaneously a straight and a flush.
 - (a) What is the probability of drawing a straight flush?
 - (b) What is the probability of drawing either a straight, a flush, or a straight flush?
 5. Consider the experiment of rolling four fair 6-sided dice. Let E be the event that the first two dice come up as a pair, and F the event that the four dice come up as two pair (e.g. 3, 5, 5, 3).
 - (a) What is the sample space for this experiment?
 - (b) What is the probability that E occurs?
 - (c) What is the probability that F occurs?
 - (d) Are E and F independent? Explain why or why not.

6. Consider the following game, played in rounds using four fair 6-sided dice. Your goal is to roll at least one 6. If you fail to do so, you may roll again, but using one fewer die (repeatedly: you keep rolling until you roll one die in the last round).
- (a) Draw a tree diagram corresponding to this game.
 - (b) What is the probability that you win in the first round?
 - (c) What is the probability that you win eventually?
 - (d) What is the conditional probability that you win eventually, given that you have not yet won and are rolling two dice?