Math 400 - Practice Exam 1

1. Consider the points with coordinates $P(2,1)$ and $Q(8,9)$.
   (a) Find the coordinates of the point R lying halfway in between P and Q.
   (b) Find an equation for the circle passing through P and Q with center R.
   (c) Find an equation for the line passing through R that is perpendicular to the line segment $\overline{PQ}$.

2. A factory manufacturing bicycles has fixed costs of \$10000 per month and a production cost of \$120 per bicycle. They sell the bicycles for \$160 each.
   (a) What is the cost function?
   (b) What is the revenue function?
   (c) What is the profit function?
   (d) What is their break-even point?
   (e) What is their profit if they sell 500 bicylces?

3. Consider the system of equations
$$\begin{aligned} x + y + 2z &= 4 \\ 2x + 2y + 6z &= 10. \end{aligned}$$
   (a) Write down the augmented matrix corresponding to this system.
   (b) Use row operations to find the row-reduced echelon form of the augmented matrix of part (a).
   (c) Does this system have a unique solution, infinitely many solutions, or no solutions? If there are solutions, write down a general form for the solution.
   (d) Write down the matrix equation corresponding to this system.
   (e) Why is it impossible to solve this system using matrix inverses?

4. Consider the system of equations
$$\begin{aligned} x + 2y &= 2 \\ y - 4z &= 4 \\ 2z &= 8 \end{aligned}$$
   (a) Express the system as a matrix equation $A\mathbf{v} = \mathbf{b}$.
   (b) Find A^{-1}.
   (c) Solve the system using the matrix inverse.

5. For what value(s) of k does the matrix
$$\begin{pmatrix} 2 & 3 \\ 4 & k \end{pmatrix}$$
   have no inverse?

6. Construct the truth table for the compound proposition $(\sim p \vee q) \wedge (p \vee q)$.

7. Which of these steps is not justified by one of the laws of logic?

$$\begin{aligned}(\sim p \vee q) \wedge (p \vee q) &\Leftrightarrow (\sim p \wedge p) \vee (q \wedge q)\\ &\Leftrightarrow c \vee (q \wedge q)\\ &\Leftrightarrow c \vee q\\ &\Leftrightarrow q\end{aligned}$$

8. Consider the argument

   If you prepare for the exam, then you will pass.
   You will pass the exam.

   Therefore, you prepared for the exam.

   (a) What are the premises and what is the conclusion?

   (b) Write the argument symbolically, and determine whether it is valid.