

## Math 400 - Exam 2

Name: \_\_\_\_\_

ID: \_\_\_\_\_

Problem 1 (8 points)	
Problem 2 (8 points)	
Problem 3 (12 points)	
Problem 4 (26 points)	
Problem 5 (14 points)	
Problem 6 (6 points)	
Problem 7 (12 points)	
Problem 8 (14 points)	
Total (100 points)	

1. (8 points) Suppose you invest \$1000 in a savings account, and you leave it there for 45 years (until you retire). If the account earns a nominal rate of 4% per year and is compounded twice a year, give an expression for the amount of money in the account when you retire?

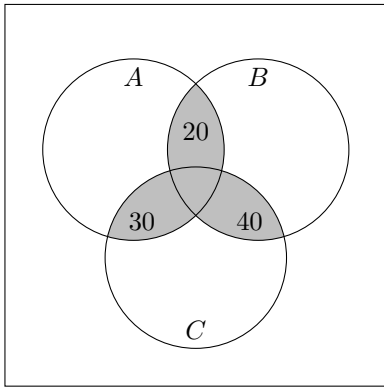
2. (8 points) How many ways are there to choose 3 apples from a pile of 6 apples, ignoring order? Give you answer as a number (rather than leaving it unsimplified).

3. Find the following sums (show your work; adding terms one by one will not receive credit)

(a) (6 points)  $5 + 9 + 13 + 17 + \cdots + 95$

(b) (6 points)  $2 + 6 + 18 + 54 + \cdots + 2 \cdot 3^7$

4. Consider the follow Venn diagram of the sets  $A$ ,  $B$  and  $C$ , where the numbers indicate the number of elements in that region.



- (a) (6 points) Give an expression in terms of  $A$ ,  $B$  and  $C$ , intersections, unions and complements that describes the shaded region (Hint: there are multiple correct answers. One possibility gives it as the union of three sets).

- (b) (10 points) Suppose that  $A$ ,  $B$  and  $C$  all contain 100 elements, and  $A \cup B \cup C$  contains 190 elements. How many elements are contained in the shaded region?

(c) (5 points) If an element of  $A \cup B \cup C$  is selected uniformly at random, what is the probability that it lies in the shaded region?

(d) (5 points) Given that an element selected lies in the shaded region, what is the probability that it lies in  $A$ ?

5. You line up five books on a shelf at random. Three of the books (titled A, B and C) are blue and two (titled D and E) are yellow.

(a) (6 points) How many different arrangements of the five books are possible?

(b) (8 points) What is the probability that all of the blue books end up to the left of the yellow books?

6. (6 points) Pennsylvania license plates have three letters and then four digits (e.g. ABA-1233). How many license PA license plates are possible?

7. Suppose that events  $A$  and  $B$  are independent with  $P(A) = 0.5$  and  $P(B) = 0.4$ .

(a) (6 points) What is the probability that both  $A$  and  $B$  occur?

(b) (6 points) What is the probability that either  $A$  or  $B$  occurs?

8. Suppose that the probability you make a mistake on your homework depends on how much sleep you got the previous night. If you got at least 8 hours of sleep, there is a 10% chance of a mistake. If you got between 6 and 8 hours of sleep, there is a 40% chance of a mistake. And if you got less than 6 hours of sleep, there is a 75% chance of a mistake. The likelihood that you get at least 8 hours of sleep is 20% and that you get between 6 and 8 hours of sleep is 50%.
- (a) (6 points) Draw a tree diagram corresponding to this setup.

- (b) (8 points) What is the probability that you make a mistake on your homework?