# Math 400 - Exam 1

Name: _____

ID: _____

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| Problem 1 (8 points) | |
| Problem 2 (14 points) | |
| Problem 3 (14 points) | |
| Problem 4 (17 points) | |
| Problem 5 (15 points) | |
| Problem 6 (8 points) | |
| Problem 7 (12 points) | |
| Problem 8 (12 points) | |
| Total (100 points) | |

1. (8 points) Find the points on the $x$-axis that are a distance of 5 from the point $(1, 4)$.

2. Let $L$ be the line passing through the points $P(1,1)$ and $Q(5,7)$ and $L'$ be the line passing through the points $P'(-3,0)$ and $Q'(0,2)$.

   (a) (6 points) Find equations for $L$ and $L'$.

   (b) (4 points) Is $L$ perpendicular to $L'$? Why or why not?

   (c) (4 points) Find the intersection of $L$ and $L'$.

3. A factory is making three kinds of items (puzzles, rattles, and abacuses) using three kinds of components (screws, beads, and dowels). It takes one screw, two beads and one dowel to make a puzzle; it takes one bead and two dowels to make a rattle; and it takes three screws, four beads and one dowel to make an abacus. At the end of the day, the manager determines that she has eleven screws, seventeen beads and seven dowels remaining. She wants to determine how many puzzles, rattles and abacuses should she make in order to use all of these materials.

   (a) (4 points) Write a system of equations whose solution will give the number of puzzles, rattles and abacuses to make.

   (b) (10 points) Use row operations to solve the system.

4. Consider the system of equations

$$\begin{aligned} x + 3y &= 1 \\ 2x - y &= 2 \\ 3x + 2y &= 3 \end{aligned}$$

(a) (2 points) Write down the augmented matrix corresponding to this system.

(b) (6 points) Use row operations to find the row-reduced echelon form of the augmented matrix of part (a).

(c) (4 points) Does this system have a unique solution, infinitely many solutions, or no solutions? If there are solutions, write down a general form for the solution.

(d) (2 points) Write down the matrix equation corresponding to this system.

(e) (3 points) Why is it impossible to solve this system using matrix inverses?

5. Let $A = \begin{bmatrix} -3 & 4 \\ 4 & -5 \end{bmatrix}$.

(a) (6 points) Find $A^{-1}$.

(b) (6 points) Use $A^{-1}$ to solve the equation

$$\begin{bmatrix} -3 & 4 \\ 4 & -5 \end{bmatrix} \cdot \left( \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

(c) (3 points) Explain why the equation

$$\begin{bmatrix} -3 & 4 \\ 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

has no solutions.

6. (8 points) Construct the truth table for the compound proposition $\sim p \wedge (q \rightarrow r)$.

7. Let $p$ be the proposition "The milk's expiration date has passed" and $q$ the proposition "The milk has gone sour."

   (a) (2 points) Express $p \to q$ in words.

   (b) (2 points) Express the contrapositive of $p \to q$ both symbolically and in words.

   (c) (2 points) Express the converse of $p \to q$ both symbolically and in words.

   (d) (2 points) Express the inverse of $p \to q$ both symbolically and in words.

   (e) (4 points) Which of these statements are logically equivalent?

8. Consider the argument $\begin{array}{l} p \wedge \sim q \\ p \rightarrow q \\ \hline p \end{array}$

(a) (4 points) What are the premises and what is the conclusion?

(b) (8 points) Determine whether this argument is valid, justifying your reasoning either with a truth table or the laws of logic.