Math 240 – Practice Exam 2

1. Evaluate
$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx.$$

2. Consider a laminate of radius 1 whose density at any point is proportional to the distance from the center. Find the moment of inertia about its center.

3. Find the limits of integration when rewriting the following integral with a different order of integration.
$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 f(x,y,z)\, dz\, dy\, dx = \int_?^? \int_?^? \int_?^? f(x,y,z)\, dx\, dy\, dz.$$

4. Evaluate the following integrals:

   (a)
$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{8-x^2-y^2}}^{-\sqrt{x^2+y^2}} xz\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

   (b)
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_{-1}^{1-\sqrt{x^2+y^2}} yz(2+\sqrt{x^2+y^2})\, dz\, dy\, dx.$$

5. Let R be the region bounded by the ellipse $2x^2 - 2xy + y^2 = 5$. Using the change of variables
$$\begin{aligned} x &= 2u + v \\ y &= u + 3v, \end{aligned}$$
   evaluate
$$\iint_R x^2\, dA.$$

6. Find the maximum and minimum values of $x^2 + 2y^2 + 2z^2$ subject to the constraint $x + y + z = 4$.