Math 240 – Practice Exam 1

1. Find the distance between the lines

$$L_1 : \langle t-1, t+1, 2t\rangle$$
$$L_2 : \langle 2t+1, 0, t-1\rangle$$

2. Consider the curve parameterized by

$$\mathbf{r}(t) = \left\langle t\sin(t), \frac{2\sqrt{2}}{3}t^{3/2}, t\cos(t)\right\rangle.$$

Find the arclenth along this curve between $(0,0,0)$ and $(0, \frac{8}{3}\pi^{3/2}, 2\pi)$.

3. Consider the curve parameterized by

$$\mathbf{r}(t) = \left\langle t^2, t^3 - t, t\right\rangle.$$

Find the curvature as a function of t.

4. Suppose that S is a surface that contains the two curves given by

$$\mathbf{r}_1(t) = \langle t\cos(t) - 1, t^2 + t + 1, 2e^t\rangle$$
$$\mathbf{r}_2(t) = \langle \ln(t) - t, \sqrt{t}, \frac{2}{t}\rangle$$

Find an equation for the tangent plane to S at the point $(-1, 1, 2)$.

5. Suppose that $f(x,y)$ has continuous second partial derivatives, and that

$$g(u,v) = f(u^2+v^2, u^2-v^2).$$

Suppose that the derivatives of $f(x,y)$ are given in the following table. Each row gives the value of f and its derivatives at a specified point (x,y).

	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
$(1,1)$	1	2	0	-1	1	3
$(2,0)$	0	-1	1	2	0	1

Find the value of $g_{uu}(1,1)$.

6. Let $f(x,y,z) = xe^{xyz}$.

(a) In what direction is $f(x, y, z)$ increasing the most rapidly near $(1, 1, 1)$? How quickly is it increasing in this direction?

(b) Find an equation for the tangent plane to $f(x, y, z) = e$ at the point $(1, 1, 1)$.

7. Let $f(x, y) = x^2 - 4xy + 2y^4$. Find the critical points of f and determine which are local minima, local maxima or saddle points.