1. We write $y = \frac{x+1}{2x+1}$ and solve for $x$:

\begin{align*}
(2x + 1)y &= x + 1 \\
2xy - x + y - 1 &= 0 \\
(2y - 1)x &= 1 - y
\end{align*}

So $x = \frac{1 - y}{2y - 1}$.

So $f^{-1}(x) = \frac{1 - x}{2x - 1}$.

3. The derivative at $x = 0$ is

$$\lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \frac{|h| \sqrt[3]{h} - |0| \sqrt[3]{0}}{h} = \lim_{h \to 0} \frac{|h|}{h} \sqrt[3]{h}.$$

When $h > 0$, $\frac{|h|}{h} = 1$ and $\sqrt[3]{h}$ tends to 0 as $h \to 0$. When $h < 0$, $\frac{|h|}{h} = -1$ and $-\sqrt[3]{h}$ also tends to 0 as $h \to 0$. Therefore the limit exists, and $f(x)$ is differentiable at 0 with $f'(0) = 0$.

4. (a) $f'(x) = 20 \cos(5x) + \frac{2}{3 \sqrt{x^3}} + \frac{2}{x}$.

(b) $y'(x) = \frac{2e^{2x}(1+x^2) - 2xe^{2x}}{(1+x^2)^2} = \frac{2(1-x+x^2)e^{2x}}{(1+x^2)^2}$.

(c) $y' = 3x^2 \tan^{-1}(4x) + \frac{4x^3}{1+16x^2}$.

(d) $y = e^{\ln(x+1)x}$ so $y' = \left(\frac{x}{x+1} + \ln(x+1)\right)(x+1)x$. You can also use logarithmic differentiation.

(e) $f'(x) = \sin(x) \frac{\sqrt{x}}{\sqrt{x+1}}$.

5. We use implicit differentiation to get

$$2x + 4y + 4xy' + 3y^2y' = 0.$$

Solving for $y'$ and substituting $x = 2$ and $y = 1$,

\begin{align*}
y'(4x + 3y^2) &= -2x - 4y \\
y'(8 + 3) &= -4 - 4 \\
y' &= -\frac{8}{11}.
\end{align*}

Using the point-slope equation of the line passing through $(2, 1)$ with slope $-\frac{8}{11}$, we get

$$y - 1 = -\frac{8}{11}(x - 2).$$
6. (a) As \( x \to \infty \), \( e^{-x} \) goes to 0 faster than \( x \) goes to infinity, so \( \lim_{x \to \infty} f(x) = 0 \). There are no other vertical or horizontal asymptotes.

(b) \( f'(x) = e^{-x} - xe^{-x} = (1 - x)e^{-x} \), which is negative when \( x > 1 \) and positive when \( x < 1 \). So \( f(x) \) is increasing for \( x < 1 \) and decreasing for \( x > 1 \).

(c) There is a critical point at \( x = 1 \), which is a local maximum by the first derivative test.

(d) Differentiating again, \( f''(x) = -e^{-x} - (1 - x)e^{-x} = (x - 2)e^{-x} \). So \( f(x) \) is concave up for \( x > 2 \) and concave down for \( x < 2 \), with an inflection point at \( x = 2 \).

(e) Using the points \((0, 0)\) and \((1, 1/e)\), we get the following graph:

7. We have \( A = LW \). Differentiating with respect to time,

\[
A' = L'W + W'L = 8 \cdot 10 + 3 \cdot 20 = 140.
\]

So the area is increasing at a rate of 140 square centimeters per second.

8. Differentiating,

\[
f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)
\]

We compute the value of \( f(x) \) at the three critical points 0, 1 and \(-1\), and at the endpoints of the interval, \(-2\) and 3.

\[
\begin{align*}
f(-2) &= 16 - 8 + 3 = 11 \\
f(-1) &= 1 - 2 + 3 = 2 \\
f(0) &= 0 - 0 + 3 = 3 \\
f(1) &= 1 - 2 + 3 = 2 \\
f(3) &= 81 - 18 + 3 = 66.
\end{align*}
\]

So the absolute minimum is 2 and the absolute maximum is 66.

9. The picture is
So the area is

\[ A = 2xy, \]

and the fact that \((x, y)\) lies on the parabola implies that \(y = 16 - x^2\). Thus

\[ A = 2x(16 - x^2). \]

Differentiating, we have

\[ 0 = 32 - 6x^2, \]

so \(x = \frac{4}{\sqrt{3}}\) and \(y = 16 - \frac{32}{3}\). So the largest rectangle has width \(2x = \frac{8}{\sqrt{3}}\) and height \(\frac{32}{3}\).

10. Draw the tangent line to the curve above \(x_1\), and \(x_2\) will be the intersection of that tangent line with the \(x\)-axis. Repeat to get \(x_3\) and \(x_4\).

11. The linear approximation near \(a = 1\) is

\[
L(x) = f(a) + f'(a)(x - a) \\
= \sqrt{a} + \frac{1}{9}a^{-8/9}(x - a) \\
= 1 + \frac{1}{9}(x - 1)
\]

Therefore

\[
\sqrt{1.1} \approx L(1.1) \\
= 1 + 0.1/9 \\
\approx 1.011111
\]

12. (a)

\[
\lim_{x\to 2^-} \frac{|x - 2|}{x^2 - 4} = \lim_{x\to 2^-} \frac{2 - x}{x^2 - 4} \\
= \lim_{x\to 2^-} \frac{-1}{x + 2} \\
= \frac{-1}{4}
\]
(b) \[
\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1} = \lim_{x \to \infty} \frac{3 - 1/x - 2/x^2}{5 + 4/x + 1/x^2} = \frac{3}{5}
\]

(c) \[
\lim_{x \to 0} \frac{\sin(6x)}{\ln(x+1)} = \lim_{x \to 0} \frac{6\cos(6x)}{1/(x+1)} = 6
\]

by L’Hospital’s rule. Note that we need to check that \(\frac{\sin(6 \cdot 0)}{\ln(0+1)} = \frac{0}{0}\), so this is an indeterminate form where L’Hospital’s rule applies.

(d) Since \[
\frac{1}{x} - \frac{1}{\sin(x)} = \frac{\sin(x) - x}{x \sin(x)}
\]

and both numerator and denominator evaluate to 0 when \(x = 0\), we may use L’Hospital’s rule. Differentiating top and bottom, we get \[
\frac{\cos(x) - 1}{\sin(x) + x \cos(x)}.
\]

Again, both numerator and denominator evaluate to 0 so we apply L’Hospital’s rule again.

\[
\frac{-\sin(x)}{2\cos(x) - x \sin(x)}
\]

evaluates to \(\frac{0}{2}\), so \[
\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{\sin(x)} \right) = 0.
\]

(e) This limit is of indeterminate form \(1^\infty\), so we need to take the logarithm and apply L’Hospital’s rule. The natural log is \[
\frac{\ln(1 + 3x)}{x},
\]

and differentiating numerator and denominator yields \[
\frac{3/(1 + 3x)}{1}.
\]

Evaluating at \(x = 0\) gives 3, so the original limit is \(e^3\).

13. We integrate to find that \(f'(t) = 2e^t - 3\cos(t) + C\), and integrate again to get \(f(t) = 2e^t - 3\sin(t) + Ct + D\). Evaluating at 0 we get \(0 = f(0) = 2 + D\), so \(D = -2\). Evaluating at \(\pi\) we get \(0 = f(\pi) = 2e^\pi + C\pi - 2\), so \(C = \frac{2-e^\pi}{\pi}\). Thus \(f(t) = 2e^t - 3\sin(t) + \frac{2-e^\pi}{\pi}t - 2\).
14. There are four intervals, with midpoints at 1.5, 2.5, 3.5 and 4.5. The relevant Riemann sum is
\[
\frac{1}{(1.5)^3 + 1} + \frac{1}{(2.5)^3 + 1} + \frac{1}{(3.5)^3 + 1} + \frac{1}{(4.5)^3 + 1}.
\]

15. (a)
\[
\int_1^9 \frac{3x - 1}{\sqrt{x}} \, dx = \int_1^9 3x^{1/2} - x^{-1/2} \, dx
= \left[ 2x^{3/2} - 2x^{1/2} \right]_1^9
= (2 \cdot 27 - 2 \cdot 3) - (2 - 2)
= 48.
\]

(b) Using substitution with \( u = 4 + t^2 \),
\[
\int_0^2 t\sqrt{4 + t^2} \, dt = \frac{1}{2} \int_4^8 \sqrt{u} \, du
= \left[ \frac{1}{3} u^{3/2} \right]_4^8
= \frac{16\sqrt{2} - 8}{3}
\]

(c) This is the area of a semicircle of radius 2, which is \( 2\pi \).

16. (a) Using substitution with \( u = \ln(x) \) and \( du = \frac{dx}{x} \),
\[
\int \frac{\ln(x)}{x} \, dx = \int u \, du
= \frac{u^2}{2} + C
= \frac{\ln(x)^2}{2} + C.
\]

(b) Using integration by parts with \( u = \ln(x) \) and \( dv = x \, dx \),
\[
\int x \ln(x) \, dx = \frac{1}{2} x^2 \ln(x) - \frac{1}{2} \int x \, dx
= \frac{1}{2} x^2 \ln(x) - \frac{1}{4} x^2 + C.
\]

(c) Using the identity \( \sin^3(x) = \sin(x)(1 - \cos^2(x)) \) and the substitution \( u = \cos(x) \),
\[
\int \sin^3(x) \, dx = \int \sin(x) - \cos^2(x) \sin(x) \, dx
= -\cos(x) + \int u^2 \, du
= -\cos(x) + \frac{1}{3} \cos^3(x) + C.
\]