Math 220 Final Exam (part 2)

Name: _____

ID: _____

Score: _____ /60

1. Evaluate the following integrals. (5 points each)

   (a)

$$\int 25x^4 \ln(x)\, dx$$

   (b)

$$\int_0^{\pi} 15 \sin^3(x) \cos^2(x)\, dx$$

(c)

$$\int \frac{4x}{1+4x^4}\,dx$$

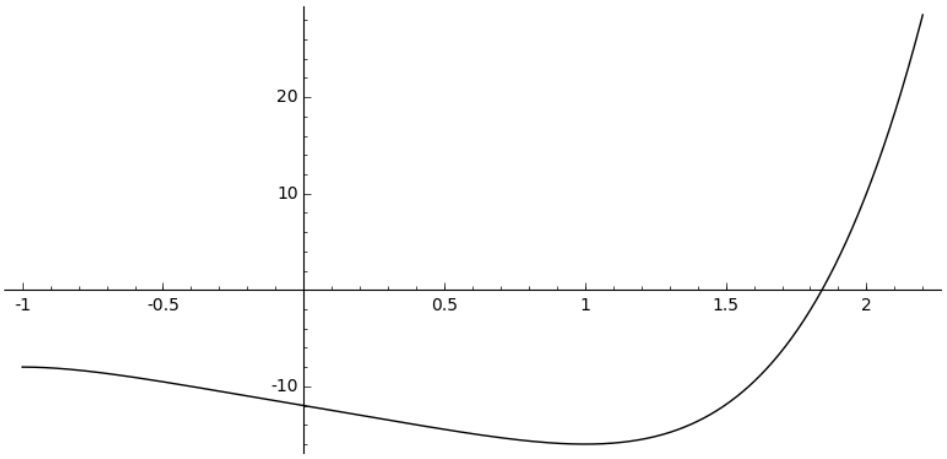
(d)

$$\int_0^1 x\sinh(x)\,dx$$

2. Suppose that a car's velocity is given by $v(t) = 60 + t\sin(t)$. Find its average velocity during the interval between $t = 0$ and $t = 10$. (10 points)

3. Consider a window formed from two panes of glass: a semicircular pane above a rectangular pane. The two panes are touching, and the diameter of the semicircle is the same as one edge of the rectangle. If the total perimeter is required to be 10 feet, what is the radius r of the semicircle that maximizes the area? (12 points)

4. Consider the function $f(x) = x^5 - 5x - 12$, whose graph is shown below.



(a) If you want to approximate the smallest positive root of $f(x)$ using Newton's method, what is a good initial estimate x_0? Note that there may be multiple acceptable answers. (1 point)

(b) On the graph above, draw the process used to find x_1, starting from your choice of x_0 in part (a). (3 points)

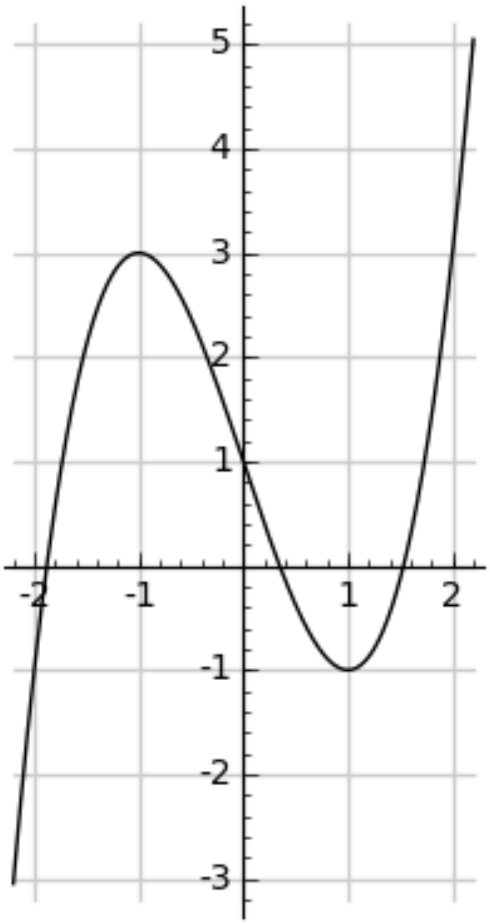
(c) Find x_1 numerically (you do not need to simplify). (4 points)

5. Let

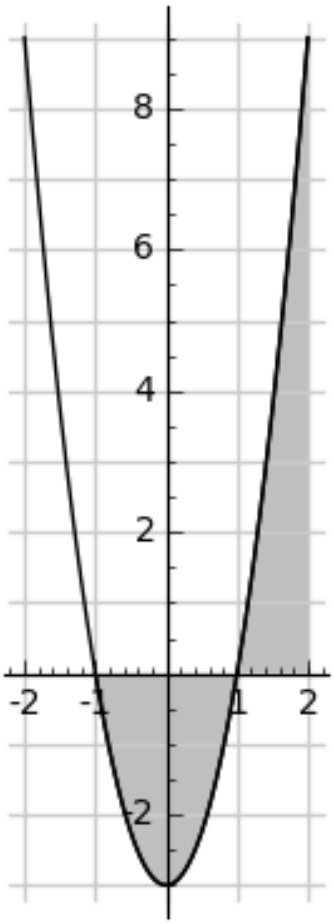
$$f(x) = \int_{-x^2}^{0} e^{t^2}\, dt.$$

Find $f'(x)$. (5 points)

6. Shown below are the graphs of a function $f(x)$ and its derivative $f'(x)$.



$f(x)$



$f'(x)$

Which of the two shaded areas is larger? **Justify your answer**. (5 points)