Math 220 Final Exam (part 1)

Name: _____

ID: _____

Score: _____ /60

1. Evaluate each of the following limits, **showing your work**. If a limit has value $\pm\infty$, give that rather than "does not exist." (3 points each)

   (a) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

   (b) $\lim_{h\to 2^+} \frac{h^2-2}{h-2}$

   (c) $\lim_{x\to 0}(1+\sin(2x))^{1/x}$

   (d) $\lim_{x\to\infty} x^2 e^{-x}\sin(x)$

2. Find the following derivatives. (3 points each)

(a) Find the derivative of

$$f(x) = \frac{1 + x^3 e^x}{1 - x^2}.$$

(b) Find the derivative of

$$g(x) = \cosh(\cos(\ln|x|)).$$

(c) Find the derivative of

$$h(x) = (2 + \sin(x))^x.$$

(d) Suppose that $y(x)$ satisfies

$$3x^2y^3 - e^y = 3 - e$$

and $y(1) = 1$. Find $y'(1)$.

3. Let

$$f(x) = \frac{1}{2}x^2 e^{1-x^2}$$

with derivatives

$$f'(x) = (x - x^3)e^{1-x^2}$$
$$f''(x) = (2x^4 - 5x^2 + 1)e^{1-x^2}$$

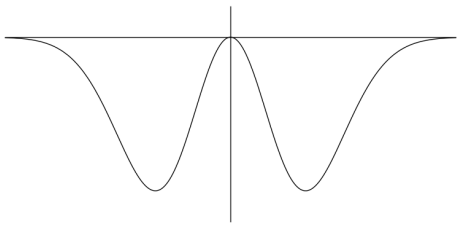
(a) Where is $f(x)$ increasing and where is it decreasing? **Show your work**. (3 points)

(b) Where is $f(x)$ concave up and where is it concave down? **Show your work**. (3 points)
Hint: The equation $2x^4 - 5x^2 + 1 = 0$ has solutions $\pm\frac{\sqrt{5\pm\sqrt{17}}}{2}$.

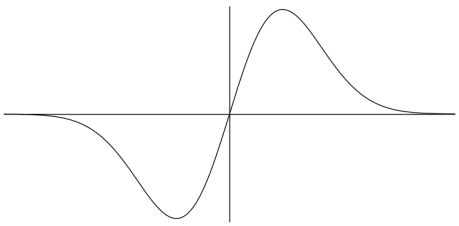
(c) Where does $f(x)$ have local maxima and local minima? **Show your work**. (3 points)

(d) Which of the following is the graph of $f(x)$? (3 points)

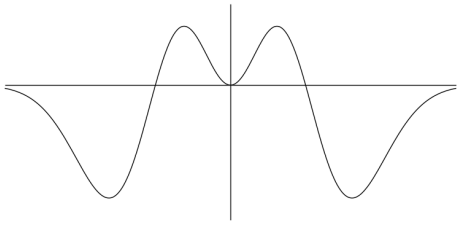
(i)



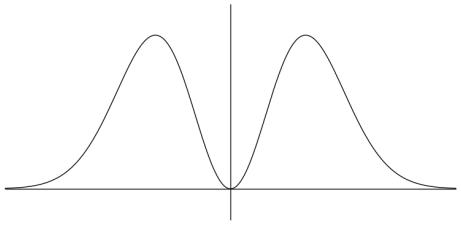
(ii)



(iii)



(iv)



4. Find the absolute minimum and maximum values of $f(x) = \frac{1}{4}x^4 - x^3 - 2x^2 + 1$ on the interval $[-1, 2]$. **Show your work**. (8 points)

5. Let $f(x) = 2x - \sin(x)$.

   (a) Find $f^{-1}(2\pi)$. **Show your work**. (4 points)

   (b) Find $(f^{-1})'(2\pi)$. **Show your work**. (4 points)

6. The volume of a cube is increasing at a constant rate of 30 cubic meters per second. When the cube has volume 1000 cubic meters, how fast is its surface area increasing? (8 points)