1. (10 pts.) Find all values of the constant c that make the function $f(x)$ continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} c^2x^2 - 3x - 1, & \text{if } x < 1 \\ 3c\cos(x-1), & \text{if } x \geq 1 \end{cases}$$

2. (10 pts.) Find the linear approximation of the function $f(x) = \sin(2x)$ at $a = \dfrac{\pi}{6}$.

3. (10 pts.) Find the derivative of the function $f(x) = \sqrt{3x-1}$ using the limit definition of the derivative. NO CREDIT will be given if Limit Definition is not used.

4. (10 pts.) Find an equation of the tangent line to the curve

$$4\sqrt{3x-y} = x\,y + 9$$

at the point $(1, -1)$.

5. (25 pts; 5 pts. each) Find the limit, if it exists. If the limit does not exist, explain why. All work must be shown.

   (a) $\displaystyle\lim_{x\to 2^+} \frac{|2-x|}{x^2 - 7x + 10}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{\sin(5x)}{x}$

   (c) $\displaystyle\lim_{x\to\infty} \frac{3 - x^3}{(2x^2+1)(x-2)}$

   (d) $\displaystyle\lim_{x\to 0} \frac{e^x - e^{2x}}{\sqrt{5x+1} - 1}$

   (e) $\displaystyle\lim_{x\to 0}(1 + \arcsin(2x))^{\frac{1}{x}}$

6. (25 pts; 5 pts. each) Differentiate the following functions. You do not have to simplify your answers.

(a) $f(x) = x^2 \cos(3x+1)$

(b) $f(x) = \dfrac{\sqrt{x}}{2\tan(x)}$

(c) $f(x) = (\tan^{-1}(3x) + 2^x)^2$

(d) $f(x) = (\sin x)^{\sqrt[3]{x}}$

(e) $f(x) = \displaystyle\int_2^{x^2} \sqrt{1+t^3}\, dt$

7. (10 pts.) Use Newton's method with the initial approximation $x_1 = 0$ to find the second approximation x_2 to the root of the equation $x - \cos(2x) = 0$.

8. (10 pts.) Find the absolute maximum and the absolute minimum of $f(x) = \dfrac{x}{x^2+1}$ on the interval $[0, 3]$.

9. (10 pts.) A bacteria culture grows with constant relative growth rate. After 2 hours, there are 400 bacteria, and after 3 hours, there are 1600 bacteria. Find the initial population. Simplify your answer as much as possible.

10. (10 pts.) The base of a triangle is increasing at a rate of 3 cm/s, and the height is decreasing at a rate of 2 cm/s. At what rate is the area of the triangle changing when the height is 10 cm and the area is 80 cm?

11. (10 pts.) A box with an open top and a rectangular base is to be constructed from 24 ft^2 of cardboard. The length of the base of the box is twice its width. Find the largest possible volume of the box.

12. (20 pts; 5 pts. each) Given the function $f(x) = x^4 e^{-x}$,

   (a) Find the critical numbers of $f(x)$.

   (b) Find the intervals on which $f(x)$ is increasing and decreasing, and the local maximum and minimum values of $f(x)$.

   (c) Find the inflection points of $f(x)$.

   (d) Find the intervals of concavity.

13. (25 pts; 5 pts. each) Evaluate the following integrals.

(a) $\int_1^4 |3-x|\,dx$

(b) $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$

(c) $\int \sin^3 x \cos^4 x\,dx$

(d) $\int \frac{1}{x(1+(\ln x)^2)}\,dx$

(e) $\int \sqrt{x}\ln x\,dx$

14. (15 pts.) Sketch the graph of the function that satisfies all of the given conditions. Indicate clearly all asymptotes, the intervals on which $f(x)$ is increasing and decreasing, local maxima and minima, the inflection points, and the intervals of concavity.

$f(x)$ is defined for all $x \neq 0$, $x \neq 3$;

$\lim_{x\to-\infty} f(x) = 1, \qquad \lim_{x\to+\infty} f(x) = 0$

$\lim_{x\to 0^-} f(x) = -\infty, \qquad \lim_{x\to 0^+} f(x) = -\infty,$

$\lim_{x\to 3^-} f(x) = +\infty, \qquad \lim_{x\to 3^+} f(x) = -\infty,$

$f'(x) > 0$ if $0 < x < 3$ or $3 < x < 5$;
$f'(x) = 0$ if $x = 5$
$f'(x) < 0$ if $x < 0$ or $x > 5$;

$f''(x) > 0$ if $1 < x < 3$ or $x > 6$;
$f''(x) = 0$ if $x = 1$ or $x = 6$;
$f''(x) < 0$ if $x < 0$, $0 < x < 1$, or $3 < x < 6$.