1. (a) Find the derivative of the function

$$f(x) = \frac{1}{x}+2\cos x+3\tan x+4\cot x+5\ln x+6e^x+\log_7 x+8^x+9\arctan x+\arcsin x$$

   (b) Let $f(x) = x^2$. Find $f'(2)$ <u>by using only the definition of the derivative.</u>

   (c) Geometrically, the definite integral $\int_a^b f(x)\,dx$ represents the area of a certain region on the $x-y$ plane that is related to the curve $y = f(x)$.
   <u>Using only this geometrical interpretation</u> fin $\displaystyle\int_0^3 (2x+1)dx$.

2. Let $\vec{a} = \langle 2, 1\rangle$ and $\vec{b} = \langle 1, 3\rangle$

   (a) Find the angle between $\vec{a}$ and $\vec{b}$

   (b) Let $P(= (1, 3)$, $Q = (-1, 5)$ and $S = (5, 7)$ be points in a plane. Find the fourth vertex of the parallelogram whose sides are $\vec{PQ}$ and $\vec{PS}$.

3. The position vector of a particle traveling on the $x - y$ plane at time t is $\vec{r}(t) = \langle t, 8t - t^2\rangle$, where t is measured in seconds and coordinates are in meters.

   (a) Find the particle's average velocity vector during the time interval $[0, 2]$.

   (b) Find the particle's velocity vector, speed, and acceleration vector at time $t = 1$.

   (c) Find a non-parametric equation describing the curve that the particle passes by.

4. Find the derivatives of the following functions.

   (a) $f(x) = (x^2 + x + 1)(x^3 - 3x^2 + x + 1)$. [no simplification for answer]

   (b) $g(x) = \dfrac{x+1}{x^2+1}$.

   (c) $p(x) = (1 + x^4)^{10}$

   (d) $q(x) = \sin\left(\dfrac{1}{xe^{2x}}\right)$

5. (a) <u>Evaluate</u> $\displaystyle\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

(b) Evaluate $\lim_{x \to 0} \frac{1 - e^x}{\sin x}$

(c) Evaluate $\lim_{x \to 1} \frac{x - 1}{|x - 1|}$

(d) Evaluate $\lim_{x \to 0} \left(\frac{1 + x}{x \cos x} - \frac{1}{x} \right)$

6. (a) Find the Riemann sum $R_4 = \sum_{i=1} 4\, f(c_i)(x_i - x_{i-1})$ for $\int_0^8 x^2\, dx$ with regular partition points $x_i = 2i$ for $i = 0, 1, 2, 3, 4$, and the middle point rule: $c_i = \frac{1}{2}(x_{i-1} + x_i)$.

(b) Evaluate the definite integral $\int_0^8 x^2\, dx$.

(c) Evaluate the indefinite integral

$$\int \left(x^2 + \frac{2}{x} + 3 \cos x + \frac{4}{\sqrt{1 - x^2}} + \frac{5}{1 + x^2} \right)$$

(d) Find the derivative of the function $F(x) = \int_0^x t^2 e^{t^2}\, dt$.

7. (a) Use a linear approximation or a differential for the function $f(x) = x^{1/3}$ at $a = 1000$ to find an approximation to $\sqrt[3]{1003} - \sqrt[3]{1000}$.

(b) Let Use the Newton's Method to find a rational number that approximates the positive root to $x^2 - 2 = 0$.

8. The derivatives of the function $f(x) = xe^{-x^2/2}$ are calculated as follows

$$f'(x) = (1 - x^2)e^{-x^2/2}, \qquad f''(x) = x(x^2 - 3)e^{-x^2/2}$$

(a) Find the intervals where f is increasing or decreasing. Also find points of local or global minimum or maximum.

(b) Find intervals where f is concave up or concave down. Also find points of inflection.

(c) Find any horizontal asymptotes.

(d) Sketch the curve of $y = f(x)$ for $-\infty < x < \infty$.

9. A box with a square base, rectangular sides, and open top must have a volume of 1000 cm^3. The material for the base costs \$4/cm^2 and that for the sides \$2/cm^2. Find the dimensions of a box that minimizes the cost of material used.

10. A swimming pool of dimension 100(m) $\times$ 200(m) and horizontal bottom is drained at a rate of 2 m^3/min. Find the rate of decreasing of the depth of the water in the pool.

11. (a) Let $q(x) = x^x$. Using the logarithmic differentiation technique, find $q'(x)$.

    (b) Let $y = y(x)$ be implicitly defined by $y^3 + xy = 1$. Using the implicit differentiation technique, find $y'(x)$.

    (c) Find the equation of the line that has slope 3 and is tangent to the curve given parametrically by $x = t^2 + 1$, $y = t^3$.