Math 0220 Sample Final 3

1a. (10 pts.) Given $f(x) = x^2 + 1$, use the definition of the derivative to show that $f'(x) = 2x$.

1b. (5 pts.) Find the equation of the line tangent to the curve of $y = x^{1/3}$ at $x = 1000$.
(**Note:** $(1000)^{1/3} = 10$.)

1c. (5 pts.) Use the tangent line in part (b) to obtain an approximation for $(1005)^{1/3}$.

1d. (5 pts.) Approximate $\sqrt{2}$ by applying Newton's Method to approximate the positive zero of the function $f(x) = x^2 - 2$ with $x_1 = 1.5$. Find x_2. (You do have to show your work.)

2a. (9 pts.) Let $y = \dfrac{ln\ x}{x^3+1}$. Find $\dfrac{dy}{dx}$.

2b. (9 pts.) Let $y = (1+2x-x^3)^{100}$. Find $\dfrac{dy}{dx}$.

2c. (9 pts.) Let $y = \arctan(x^2+3)$. Find $\dfrac{dy}{dx}$.

2d. (9 pts.) Let $y = x^{\sin\ x}$. Find $\dfrac{dy}{dx}$.

2e. (9 pts.) Given $y^3 + xy + e^{2x} = 2$, find $\dfrac{dy}{dx}$ at $(0,1)$.

3. Let $f(x) = 2x^3 - 3x^2 - 12x + 5, \ -\infty < x \leq 4$. Then $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$.

3a. (5 pts.) Find the intervals on which $f(x)$ is increasing or decreasing.

3b. (5 pts.) Find the local maxima and local minima of $f(x)$.

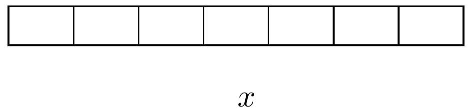
3c. (5 pts.) Find the intervals on which the graph of $f(x)$ is concave upward or concave downward.

3d. (5 pts.) Find the points of inflection.

3e. (5 pts.) Sketch the graph of $f(x)$ on $(-\infty, 4]$.

3f. (5 pts.) Find the global (absolute) maximum and the global (absolute) minimum.

4. (10 pts.) The owner of a nursery center wants to fence in 1600 square feet of land in a rectangular plot to be used for different shrubs. The plot is to be divided as follows:



What is the least number of feet of fence needed?

5a. (5 pts.) Find $\lim_{x \to \infty} \dfrac{3x^5 + x - 1}{5x^5 + 3x^2 + 2}$.

5b. (5 pts.) $\lim_{x \to 0} \dfrac{\sin(8x)}{56x}$.

5c. (5 pts.) $\lim_{t \to 0} (1 + 3t)^{\frac{1}{t}}$.

6. Let $\vec{v} = \langle -1, 4 \rangle$ and $\vec{u} = \langle 2, -3 \rangle$ be two vectors in a plane.

6a. (5 pts.) Find $||3\vec{v} - 2\vec{u}||$.

6b. (5 pts.) Find the unit vector in the direction of $\vec{u}$.

6c. (5 pts.) Find the vector of $\vec{v}$ projected on the vector $\vec{u}$.

6d. (5 pts.) Let $w = \langle t^2, 9 \rangle$. Find t such that $\vec{w}$ is perpendicular to $\vec{v}$.

7. The position vector of a particle traveling on the $x - y$ plane at time t is

$$\vec{r}(t) = \langle t,\ 4t - t^2 \rangle,\ 0 \le t \le 4$$

where t is measured in seconds and the distance is measured in meters.

7a. (5 pts.) Find the average velocity of the particle during the time from $t = 0$ to $t = 2$.

7b. (5 pts.) Find the velocity and speed of the particle at $t = 1$.

7c. (5 pts.) Find the acceleration of the particle at $t = 1$.

7d. (5 pts.) Find the equation (by eliminating t) in x and y which describes the path curve of the particle and sketch the path curve from $t = 0$ to $t = 4$.

8a. (5 pts.) Find the Riemann sum $R_4 = \sum_{i=1}^{4} f(c_i)(x_i - x_{i-1})$ for $\int_0^8 x^2 dx$ with regular partition points. $x_i = 2i$ for $i = 0, 1, 2, 3, 4$ and the midpoint rule $c_i = \frac{1}{2}(x_{i-1} + x_i)$ for $i = 0, 1, 2, 3, 4$.

8b. (6 pts.) State the definition of $\int_a^b f(x)dx$. (You do have to explain your notations.)

8c. (6 pts.) $\int_0^2 (3x^2 + \sqrt{x})\ dx.$

8d. (6 pts.) $\int \sin^2 x \ \cos\ x\ dx$

8e. (6 pts.) $\int \frac{x}{4 + x^2}\ dx$

8f. (6 pts.) $\int \frac{1}{4 + x^2}\ dx$