Math 0220 Sample Final 2

1. Let $\vec{a} = \langle 2, 1 \rangle$ and $\vec{b} = \langle 1, 3 \rangle$.

(3 pts.)

1a. Evaluate $|\vec{a} + \vec{b}|$

(3 pts.)

1b. Find the unit vector in the direction of $\vec{b}$.

(4 pts.)

1c. Find all values of t such that $\vec{a}$ is perpendicular to $\vec{c} = \langle -4, 8t \rangle$.

(5 pts.)

2a. Give a parametric vector equation for a circle of radius 9 with the center at the point $(1, -2)$.

(5 pts.)

2b. The trajectory of an object is determined by

$$\vec{r}(t) = \langle 2t, -2t^2 + 16t \rangle \text{ where } \quad -\infty < t < \infty.$$

Eliminate the parameter t and find an equation in x and y that describes the curve on which the object moves.

3. Let $f(x) = x(x-1)^2, -\infty < x < \infty$.

(10 pts.)

3a. Find all points where f has a local maximum or local minimum. Justify your answers.

(10 pts.)

3b. Find all inflection points. Justify your answer.

(10 pts.)

3c. Graph the function.

(10 pts.)

4. Find x_2, the second iterate in Newton's method, to find an approximate value for the negative solution of $x^4 = 10100$. Assume that $x_1 = -10$. Show all details.

(10 pts.)

5a. Find the equation for the line tangent to $y = x^{1/4}$ at $x = 10000$. **Hint:** $(10000)^{1/4} = 10$.

(5 pts.)

5b. Use the tangent line found in part (a) to obtain an approximate value for $(10100)^{1/4}$.

(10 pts.)

6a. Evaluate: $\lim_{x \to 0} \quad \dfrac{x}{\sqrt{x+4}-2}$

(10 pts.)

6b. Evaluate: $\lim_{x \to 3^-} \quad \dfrac{|x-3|}{x-3}$

(10 pts.)

6c. Evaluate: $\lim_{x \to 1} \quad \dfrac{\arctan(\tan(2x-3))}{2x-5}$

(10 pts.)

6d. Evaluate: $\lim_{x \to -\infty} \quad \dfrac{\ln\left(1+\frac{3}{x^2}\right)}{\sin\left(\frac{4}{x^2}\right)}$

(10 pts.)

6e. Evaluate: $\lim_{x \to 0} \quad x^2 \ln(x^2)$

(10 pts.)

7a. Evaluate: $\int \frac{dx}{4+25x^2}$

(10 pts.)

7b. Evaluate: $\int (12^x + x^{1/2})dx$

(10 pts.)

7c. Let $f(x) = \int_0^{2x} \frac{dt}{\sqrt{1+t^2}}$. Find $\frac{df}{dx}$.

(10 pts.)

8a. Find $\dfrac{dy}{dx}$ at the point $(x, y) = (0, 1)$ on the curve defined by the equation $y^2 + xe^{y^2} = 1$.

(10 pts.)

8b. Let $y = \arctan(3\sin^2(x))$. Find $y'\left(\dfrac{\pi}{4}\right)$.

(10 pts.)

8c. Let $y = x^{(2x)}$. Find $\dfrac{dy}{dx}$.

(15 pts.)

9. A particle moves along the curve $2x^2 - xy + 3y^2 = 24$. If at a given time, the particle is at position $(-3, 1)$ and the x coordinate of its velocity at this point is 5 then find the y coordinate of the velocity.