Math 0220 Sample Final 1

(10 pts.)

1a. A particle moves with speed 2 around a circle of radius 4 centered at $(x, y) = (1, 0)$. Assume that the particle is at $(x, y) = (5, 0)$ at time $t = 0$. Find the vector equation describing the motion of the particle if it moves clockwise around the circle as t increases.

(15 pts.)

1b. The trajectory of an object is described by the vector function

$$\bar{r} = (4 + 7t^3)\bar{i} + (1 - 2t)\bar{j}, \quad -\infty < t < \infty$$

Eliminate t and find an equation in x and y that describes the curve on which the object moves.

(15 pts.)

2. Use a tangent line to the function $f(x) = (8x)^{1/3}$ to find an approximate value for $(8.08)^{1/3}$.

(10 pts.)

3. Using Newton's method, find x_2, the second iterate, to approximate the solution of $x^5 + x^3 = 1$. Assume that $x_1 = 1$.

4. Given the function:

$$f(x) = \begin{cases} \dfrac{1}{x^2}, & -\infty < x \leq -1 \\ -\dfrac{x}{2}, & -1 < x \leq 0 \\ 1 + x^2, & x > 0 \end{cases}$$

Determine:

(5 pts. each)

4a. $\lim_{x \to -1^+} f(x)$

4b. $\lim_{x \to 0^-} f'(x)$

4c. Sketch the graph of the function.

(6 pts. each)

5. Find the first derivative of the following functions:

5a. $f(x) = \tan^{-1}(x^3 + 2x)$

5b. $s(x) = \sin^2(x) - \dfrac{3}{x^{1/3}}, \;\; x \neq 0.$

5c. $y = \dfrac{x}{\ln(x)}.$

5d. $h(x) = 3^{\tan(x)}.$

5e. $y = x^3 \;\ln(x^2)$

(6 pts. each)

6. Determine the following limits:

6a. $\lim_{h \to 0^+} \frac{|-2+h| - |-2|}{h}$

6b. $\lim_{x \to 0} \frac{\tan^{-1}(2+x) - \tan^{-1}(2)}{x}.$

6c. $\lim_{x \to 0^+} x^2 \ln(x^3)$

6d. $\lim_{t \to 0}(1+3t)^{\frac{1}{t}}$

6e. $\lim_{x \to 0} x^2(1-\cos(2x))^{-1}$

(15 pts.)

7. Find the equation for the line tangent to the graph of the equation $\sqrt{y+x} - \sqrt{y-x} = 2$ at $Q = (10, 26)$.

(15 pts.)

8. A spherically shaped balloon is being inflated by pumped air. The area of its surface is $S = 4\pi r^2$ square inches, and its volume is $V = \dfrac{4}{3}\pi r^3$ cubic inches, where r is the radial distance from the center of the balloon to its surface. As air is pumped into the balloon, assume that the area of the surface is increasing at a rate of 8 square inches per second. How fast is its radius increasing when the volume reaches $\dfrac{32\pi}{3}$ cubic inches.

(15 pts.)

9. A wire 16 feet long has to be formed into a rectangle. What dimensions should the rectangle have to maximize its area?

(10 pts. each)

10a. Find the area under the curve: $y = 2^x$ between $x = 0$ and $x = 5$.

10b. Evaluate $\int \frac{(x+1)}{1+2x^2} dx$

11. Consider the function $f = x^2 e^{-x}$ where $-\infty < x < \infty$.

(7 pts.)

11a. Find all values of x where f attains a relative maximum or a relative minimum. Justify your answer.

(3 pts.)

11b. Sketch the graph of the function.