Math 220 - Practice Exam 1 (version B)

1. Give a value for each of the following limits. (4 points each)

   (a) $\displaystyle\lim_{x \to 1} \frac{\sqrt{x^2 + 3} - \sqrt{3x^2 + 1}}{x - 1}$

   (b) $\displaystyle\lim_{x \to 2^+} \frac{|1 - x^2|}{x - 2}$

   (c) $\displaystyle\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2 - t} \right)$

2. Determine the derivatives of the following functions. (4 points each)

   (a) $f(x) = \sqrt[3]{x} + (1 + x)^{99}$

   (b) $f(x) = (x^3 + 1)^6 \sin(x)$

(c) $f(x) = \dfrac{x^3 + x}{3x^2 - 1}$

(d) $f(x) = \tan(\cos(x^2))$

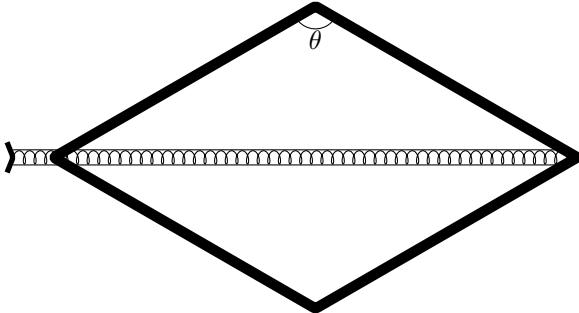(e) $f(x) = \dfrac{1}{x + \sin^2(x + x^2)}$

3. Suppose that $\displaystyle\lim_{x \to 1} \dfrac{f(x) - 4}{x - 1} = 9$. Find $\displaystyle\lim_{x \to 1} f(x)$. Justify your answer. (8 points)

4. Determine the equation of the tangent line to the curve

$$x \sin(y) - x^2 \cos(y) = 1$$

at the point $(1, \pi/2)$. (10 points)

5. A diamond shaped car jack is tightened, pulling the left and right corners together at a rate of 1mm/s.

Suppose that all sides of the jack are 300mm long. Find the rate at which the car is raised when $\theta = 2\pi/3$. Feel free to leave square roots in your answer. (10 points)

6. Determine where the function $f(x) = \frac{x^2 - x}{2x^2 - 1}$ has a horizontal tangent line. (8 points)

7. Suppose that $f(x)$ is a differentiable function with $f(1) = 8$ and $f'(1) = -3$. Let $h(x) = \sqrt{1 + f(x^2)}$. Find $h'(1)$. (10 points)
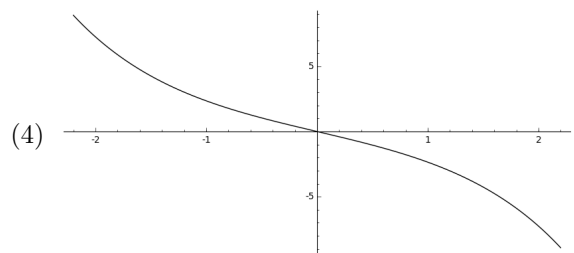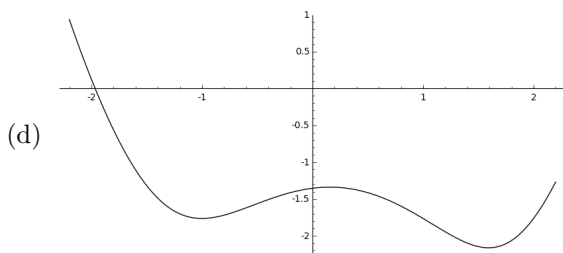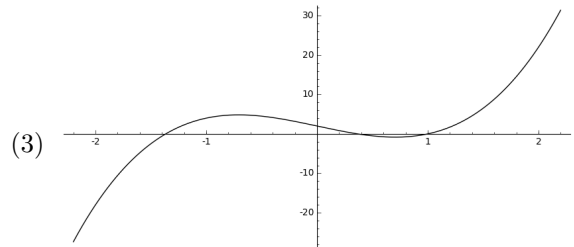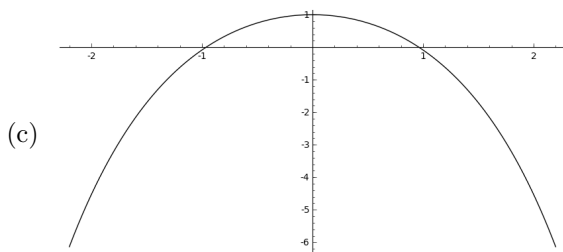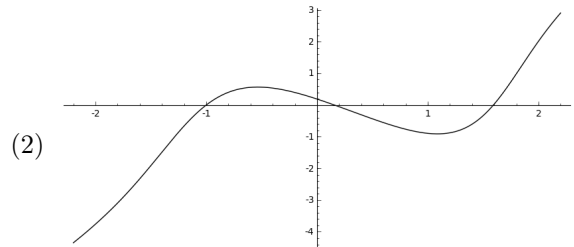
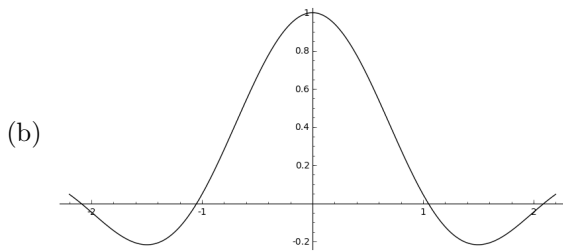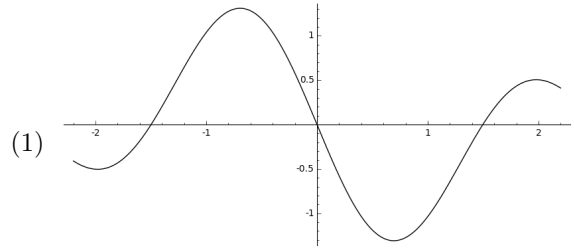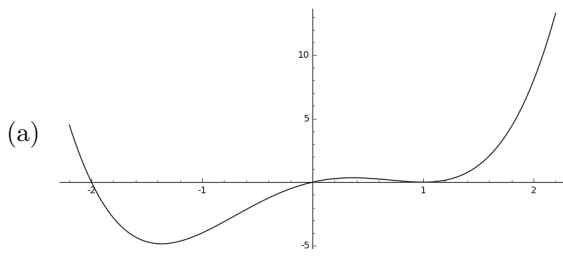8. Let $f(x) = \sqrt[3]{x}$.

    (a) Find a linear approximation to $f(x)$ near $x = a$. (5 points)

    (b) Approximate $\sqrt[3]{8.012}$. (5 points)

9. Match each graph with its derivative. (3 points per correct match)

$f(x)$

$f'(x)$

(a)

(1)

(b)

(2)

(c)

(3)

(d)

(4)

(a)

(b)

(c)

(d)