Math 220 (7:30pm section) - Exam 1

Name: ______________________________

ID: ______________________________

Score: ______________________________ /100

1. Give a value for each of the following limits (including ∞ or $-\infty$ if applicable). (4 points each)

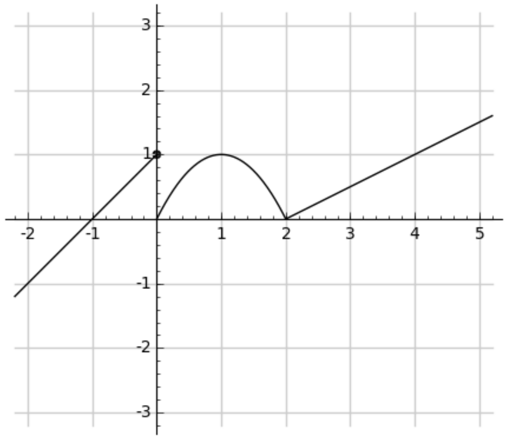
   (a) $\displaystyle\lim_{x\to 2} \frac{x^2 - 2x}{\sqrt{x+2} - \sqrt{2x}}$

   (b) $\displaystyle\lim_{\theta\to \pi^-} \cot(\theta)$

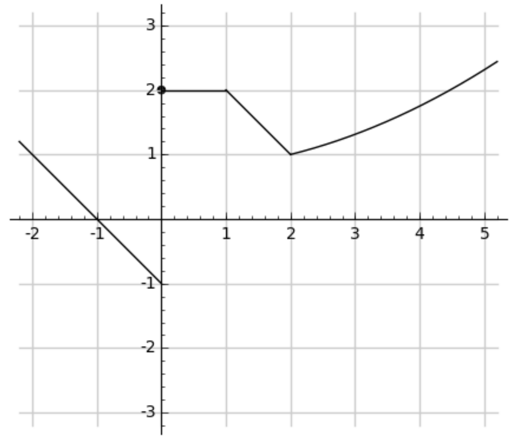
   (c) $\displaystyle\lim_{t\to 0} \left(\frac{1}{t^2 - t} + \frac{1}{t^2 + t}\right)$

2. Consider the functions $f(x)$ and $g(x)$ shown below:

$f(x)$



$g(x)$



(a) Find $\lim_{x\to 2} g(f(x))$. (4 points)

(b) Where is $f(x)$ continuous? (1 point)

(c) Where is $g(x)$ continuous? (1 point)

(d) Where is $g(f(x))$ continuous? Justify your answer. (6 points)

3. Determine the derivatives of the following functions. (4 points each)

(a) $f(x) = (\sin((1+x)^7))^3$

(b) $f(x) = x\sin(x)\sqrt[3]{1+x}$

(c) $f(x) = \dfrac{x+\tan(x)}{1-\cos^2(x)}$

(d) $f(x) = x^2 \sin\left(\dfrac{1}{x}\right)$

(e) $f(x) = (x^3+2)^5(x^2-2)^{-7/2}$

4. Find an equation for the tangent line to the curve

$$y\cos(x) = 2 + \sin(xy)$$

at the point $(0, 2)$. (10 points)

5. A balloon is being filled with air at a rate of $100\,\frac{\mathrm{cm}^3}{\mathrm{s}}$. Assuming that the balloon is spherical, how fast is its surface area increasing when its radius is $5\,\mathrm{cm}$? (10 points)

6. Determine the tangent lines to the function $f(x) = \frac{x^2+3x+4}{x-1}$ with slope -1. (10 points)

7. Suppose that $f(x)$ is a differentiable function, and $h(x) = \sqrt{1 - f(x)}$. If $h(1) = 2$ and $h'(1) = -4$, find $f'(1)$. (10 points)

8. Let $f(x) = x^9$.

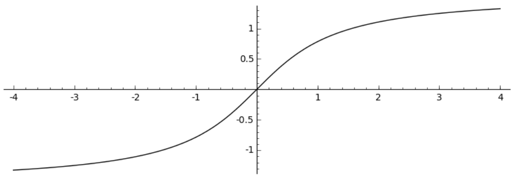
(a) Find a linear approximation to $f(x)$ near $x = a$. (4 points)

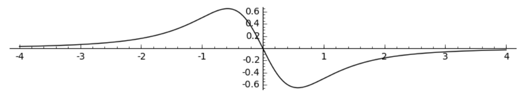
(b) Approximate $(1.01)^9$. (4 points)

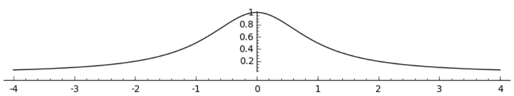
9. Match each graph with its derivative. (2 points per correct match)

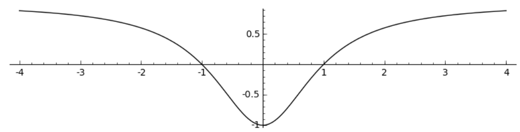
$f(x)$

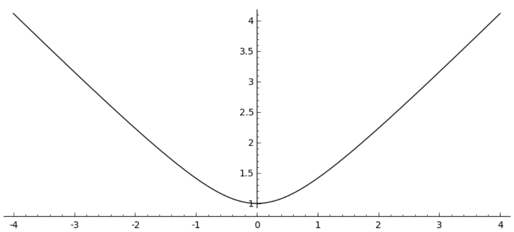
$f'(x)$

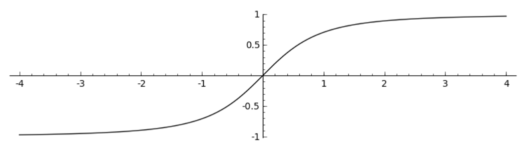
(a) 

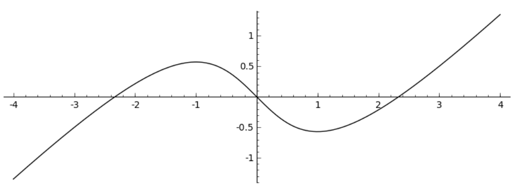
(1) 

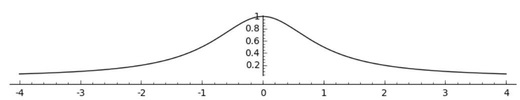
(b) 

(2) 

(c) 

(3) 

(d) 

(4) 

(a)

(b)

(c)

(d)