Math 220 - Fall 2010 Exam 2 Solutions

1. (a) $f'(x) = 2xe^{-3x} - 3x^2e^{-3x} + 3\ln(5)5^{3x} - \frac{6(1-6\sin(2x))}{x+3\cos(2x)}$
   (b) $y' = \frac{1}{1+49x^2} - \frac{6}{\sqrt{1-4x^2}}$
   (c) $g'(x) = 6\sinh^2(2x)\cosh(2x) + 9\sinh(3x)$
   (d) $h'(x) = \frac{-54e^{-2x}}{(1+9e^{-2x})^2}$

3. (a) Evaluating top and bottom at 1 gives 0/0 so L'Hospital's Rule applies. Differentiating top and bottom gives
$$\frac{-1+1/x}{-\pi\sin(\pi x)}.$$
Again, evaluating gives 0/0, so we differentiate again, giving
$$\frac{1/x^2}{\pi^2\cos(\pi x)}.$$
Now evaluating gives the answer, $-\frac{1}{\pi^2}$.
   (b) Taking the natural logarithm gives
$$\frac{\ln(x+\ln(x))}{x-1},$$
and evaluating gives 0/0. Differentiating top and bottom, we get
$$\frac{\frac{1+1/x}{x+\ln(x)}}{1},$$
which evaluates to 2. Therefore the original limit is
$$\lim_{x\to 1}(x+\ln(x))^{\frac{1}{x-1}} = e^2.$$

4. We need to find the minimum and maximum values to find the range.
$$f'(x) = 4x^3 - 12x^2 - 16x = 4x(x^2-3x-4) = 4x(x-4)(x+1).$$
Thus the only critical point in the interval is $x = 4$, and the values of $f(x)$ at the endpoints and this critical point are $f(1) = -3$, $f(4) = -120$ and $f(5) = -67$. Therefore the range is the closed interval $[-120, -3]$.

5. We have that $y = \frac{45}{x}$ and we seek to minimize $5x + 4y = 5x + \frac{180}{x}$. Differentiating,
$$5 - \frac{180}{x^2} = 0$$
$$x^2 = 36$$
$$x = \pm 6$$
The problem asked for a positive value of x, so we have $x = 6$ and $y = \frac{45}{6} = 7.5$.

6. There are vertical asymptotes at $x = \pm\sqrt{3}$. There are no horizontal asymptotes since the degree of the numerator is greater than the degree of the denominator.

   We have $f(0) = 0$, and this is the only x-intercept (and the unique y-intercept, as normal).

   The derivative vanishes at -3, at 0 and at 3, and these are a maximum, neither min nor max, minimum respectively (by the first derivative test). The graph of $f(x)$ increases to a local maximum at $(-3, -18)$ the descends to $-\infty$ as $x \to -\sqrt{3}$ from below, and descending from $+\infty$ as x increases above $-\sqrt{3}$. There is a horizontal tangent line at $x = 0$ (through the axis-intercepts at $(0, 0)$), and then $f(x)$ continues to decrease to $-\infty$ as $x \to +\sqrt{3}$. It then decends from $+\infty$ as x increases above $+\sqrt{3}$, decreasing to a local minimum at $(3, 18)$ and then $f(x) \to \infty$ as $x \to \infty$.