1. (a) \( f'(x) = 2xe^{-3x} - 3x^2e^{-3x} + 3\ln(5)5^{3x} - \frac{6(1-6\sin(2x))}{x + 3\cos(2x)} \)

(b) \( y' = \frac{1}{1+49x^2} - \frac{6}{\sqrt{1-4x^2}} \)

(c) \( g'(x) = 6\sinh^2(2x)\cosh(2x) + 9\sinh(3x) \)

(d) \( h'(x) = \frac{-54e^{-2x}}{(1+9e^{-2x})^2} \)

3. (a) Evaluating top and bottom at 1 gives 0/0 so L'Hospital's Rule applies. Differentiating top and bottom gives

\( \frac{-1 + 1/x}{-\pi \sin(\pi x)} \).

Again, evaluating gives 0/0, so we differentiate again, giving

\( \frac{1/x^2}{\pi^2 \cos(\pi x)} \).

Now evaluating gives the answer, \(-\frac{1}{\pi^2} \).

(b) Taking the natural logarithm gives

\( \frac{\ln(x + \ln(x))}{x - 1} \),

and evaluating gives 0/0. Differentiating top and bottom, we get

\( \frac{1+1/x}{x+\ln(x)} \cdot \frac{1}{1} \),

which evaluates to 2. Therefore the original limit is

\( \lim_{x \to 1} (x + \ln(x))^{\frac{1}{x-1}} = e^2 \).

4. We need to find the minimum and maximum values to find the range.

\( f'(x) = 4x^3 - 12x^2 - 16x = 4x(x^2 - 3x - 4) = 4x(x - 4)(x + 1) \).

Thus the only critical point in the interval is \( x = 4 \), and the values of \( f(x) \) at the endpoints and this critical point are \( f(1) = -3 \), \( f(4) = -120 \) and \( f(5) = -67 \). Therefore the range is the closed interval \([-120, -3]\).

5. We have that \( y = \frac{45}{x} \) and we seek to minimize \( 5x + 4y = 5x + 180/x \). Differentiating,

\( 5 - \frac{180}{x^2} = 0 \)

\( x^2 = 36 \)

\( x = \pm 6 \)

The problem asked for a positive value of \( x \), so we have \( x = 6 \) and \( y = \frac{45}{6} = 7.5 \).
6. There are vertical asymptotes at \( x = \pm \sqrt{3} \). There are no horizontal asymptotes since the degree of
the numerator is greater than the degree of the denominator.

We have \( f(0) = 0 \), and this is the only \( x \)-intercept (and the unique \( y \)-intercept, as normal).

The derivative vanishes at \(-3\), at 0 and at 3, and these are a maximum, neither min nor max, minimum
respectively (by the first derivative test). The graph of \( f(x) \) increases to a local maximum at \((-3, -18)\)
the descends to \(-\infty\) as \( x \rightarrow -\sqrt{3} \) from below, and descending from \( +\infty \) as \( x \) increases above \(-\sqrt{3}\).

There is a horizontal tangent line at \( x = 0 \) (through the axis-intercepts at \((0,0)\)), and then \( f(x) \)
continues to decrease to \(-\infty\) as \( x \rightarrow +\sqrt{3} \). It then descends from \( +\infty \) as \( x \) increases above \(+\sqrt{3}\),
decreasing to a local minimum at \((3, 18)\) and then \( f(x) \rightarrow \infty \) as \( x \rightarrow \infty \).