§ 4.1 – Vectors and Lines
Scalar quantities vs. Vector quantities

Scalar quantities have only magnitude; e.g. time, temperature.

Vector quantities have both magnitude and direction; e.g. force, wind velocity.

Two vector quantities are equal if and only if they have the same magnitude and direction.
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Two vector quantities are equal if and only if they have the same magnitude and direction.
Geometric Vectors

Let $A$ and $B$ be two points in $\mathbb{R}^2$ or $\mathbb{R}^3$.

- $\overrightarrow{AB}$ is the geometric vector from $A$ to $B$.
- $A$ is the tail of $\overrightarrow{AB}$.
- $B$ is the tip of $\overrightarrow{AB}$.
- the magnitude of $\overrightarrow{AB}$ is its length, and is denoted $||\overrightarrow{AB}||$. 

\begin{itemize}
  \item $\overrightarrow{AB}$ is the geometric vector from $A$ to $B$.
  \item $A$ is the tail of $\overrightarrow{AB}$.
  \item $B$ is the tip of $\overrightarrow{AB}$.
  \item the magnitude of $\overrightarrow{AB}$ is its length, and is denoted $||\overrightarrow{AB}||$.
\end{itemize}
• $\overrightarrow{AB}$ is the vector from $A(1, 0)$ to $B(2, 2)$.

• $\overrightarrow{CD}$ is the vector from $C(-1, -1)$ to $D(0, 1)$.

• $\overrightarrow{AB} = \overrightarrow{CD}$ because the vectors have the same length and direction.
Definition

A vector is in **standard position** if its tail is at the origin.

We co-ordinatize vectors by putting them in standard position, and then identifying them with their tips.

Thus $\vec{AB} = \vec{0P}$ where $P = P(1, 2)$, and we write $\vec{0P} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \vec{AB}$.

$\vec{0P}$ is the **position vector** for $P(1, 2)$. 
More generally, if $P(x, y, z)$ is a point in $\mathbb{R}^3$, then $\overrightarrow{0P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is the position vector for $P$. 
More generally, if \( P(x, y, z) \) is a point in \( \mathbb{R}^3 \), then \( \overrightarrow{0P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) is the position vector for \( P \).

If we aren’t concerned with the locations of the tail and tip, we simply write \( \vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \).
Theorem (§4.1 Theorem 1)

Let \( \vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) and \( \vec{w} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \) be vectors in \( \mathbb{R}^3 \). Then

1. \( \vec{v} = \vec{w} \) if and only if \( x = x_1, \ y = y_1, \ z = z_1 \).

2. \( ||\vec{v}|| = \sqrt{x^2 + y^2 + z^2} \).

3. \( \vec{v} = \vec{0} \) if and only if \( ||\vec{v}|| = 0 \).

4. For any scalar \( a \), \( ||a\vec{v}|| = |a| ||\vec{v}|| \).

Analogous results hold for \( \vec{v}, \vec{w} \in \mathbb{R}^2 \), i.e., \( \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} \), \( \vec{w} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \).

In this case, \( ||\vec{v}|| = \sqrt{x^2 + y^2} \).
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$\vec{v} = \vec{w}$ if and only if $x = x_1$, $y = y_1$, and $z = z_1$. 

$||\vec{v}|| = \sqrt{x^2 + y^2 + z^2}$. 

$\vec{v} = \vec{0}$ if and only if $||\vec{v}|| = 0$. 

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\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}.
\]

In this case, \( ||\vec{v}|| = \sqrt{x^2 + y^2} \).
Example

Let $\vec{p} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, $\vec{q} = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$, and $-2\vec{q} = \begin{bmatrix} -6 \\ 2 \\ 4 \end{bmatrix}$,
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Then

$$||\vec{p}|| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = 5,$$

$$||\vec{q}|| = \sqrt{(3)^2 + (-1)^2 + (-2)^2} = \sqrt{9 + 1 + 4} = \sqrt{14},$$

$$||-2\vec{q}|| = \sqrt{(-6)^2 + 2^2 + 4^2} = \sqrt{36 + 4 + 16} = \sqrt{56} = 2\sqrt{14}.$$
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$$= \sqrt{36 + 4 + 16}$$

$$= \sqrt{56} = \sqrt{4 \times 14}$$

$$= 2\sqrt{14} = 2||\vec{q}||.$$
**Intrinsic Description of Vectors**

Vector equality: same length and direction.

\[ \vec{0} \]: the vector with length zero and no direction.

Scalar multiplication:
- If \( \vec{v} \neq \vec{0} \) and \( a \in \mathbb{R} \), \( a \neq 0 \), then \( a \vec{v} \) has length \( |a| \cdot ||\vec{v}|| \) and
  - the same direction as \( \vec{v} \) if \( a > 0 \);
  - direction opposite to \( \vec{v} \) if \( a < 0 \).

Addition:
- \( \vec{u} + \vec{v} \) is the diagonal of the parallelogram defined by \( \vec{u} \) and \( \vec{v} \), and having the same tail as \( \vec{u} \) and \( \vec{v} \).
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If we have a coordinate system, then:

- **Vector equality:** \( \vec{u} = \vec{v} \) if and only if \( \vec{u} \) and \( \vec{v} \) are equal as matrices.

- **Scalar multiplication:** \( a \vec{v} \) is obtained from \( \vec{v} \) by multiplying each entry of \( \vec{v} \) by \( a \) (matrix scalar multiplication).

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Tip-to-Tail Method for Vector Addition

For points $A$, $B$ and $C$,

$$\vec{AB} + \vec{BC} = \vec{AC}.$$
Example (§4.1 Example 2)

Show that the diagonals of any parallelogram bisect each other.

Denote the parallelogram by its vertices, \( ABCD \).

Let \( M \) denote the midpoint of \( \overrightarrow{AC} \).

Then \( \overrightarrow{AM} = \overrightarrow{MC} \).

It now suffices to show that \( \overrightarrow{BM} = \overrightarrow{MD} \).

\[
\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AM} = \overrightarrow{CD} + \overrightarrow{MC} = \overrightarrow{MC} + \overrightarrow{CD} = \overrightarrow{MD}.
\]

Since \( \overrightarrow{BM} = \overrightarrow{MD} \), these vectors have the same magnitude and direction, implying that \( M \) is the midpoint of \( \overrightarrow{BD} \).

Therefore, the diagonals of \( ABCD \) bisect each other.
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Let $M$ denote the midpoint of $\overrightarrow{AC}$. Then $\overrightarrow{AM} = \overrightarrow{MC}$.

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[Diagram of parallelogram ABCD with points A, B, C, D and M labeled]
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Therefore, the diagonals of \(ABCD\) bisect each other.
Vector Subtraction

If we have a coordinate system, then subtract the vectors as you would subtract matrices.

For the intrinsic description:

\[ \vec{v} - \vec{u} = \vec{u} + (-\vec{v}) \]

and is the diagonal from the tip of \( \vec{v} \) to the tip of \( \vec{u} \) in the parallelogram defined by \( \vec{u} \) and \( \vec{v} \).
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Theorem ($\S$4.1 Theorem 3)

Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be two points. Then
Theorem (§4.1 Theorem 3)

Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be two points. Then

$$\overrightarrow{P_1P_2} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix}. $$
Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be two points. Then

1. $\overrightarrow{P_1P_2} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix}$.

2. The distance between $P_1$ and $P_2$ is

$$
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.
$$
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\]

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\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.
\]

Proof.

\[
0\overrightarrow{P_1} + \overrightarrow{P_1P_2} = 0\overrightarrow{P_2}, 
\]

so $\overrightarrow{P_1P_2} = 0\overrightarrow{P_2} - 0\overrightarrow{P_1}$,

and the distance between $P_1$ and $P_2$ is $||\overrightarrow{P_1P_2}||$.
Example

For $P(1, -1, 3)$ and $Q(3, 1, 0)$

$$
\overrightarrow{PQ} = \begin{bmatrix}
3 - 1 \\
1 - (-1) \\
0 - 3
\end{bmatrix} = \begin{bmatrix}
2 \\
2 \\
-3
\end{bmatrix}
$$

and the distance between $P$ and $Q$ is $||\overrightarrow{PQ}|| = \sqrt{2^2 + 2^2 + (-3)^2} = \sqrt{17}$. 

Definition

A unit vector is a vector of length one.

Example

$$
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix},
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix},
\begin{bmatrix}
\sqrt{2} \\
\sqrt{2} \\
0
\end{bmatrix}
$$

are examples of unit vectors.
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For $P(1, -1, 3)$ and $Q(3, 1, 0)$

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\]

Definition

A unit vector is a vector of length one.

Example

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}, \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}, \begin{bmatrix}
\frac{\sqrt{2}}{2} \\
0 \\
\frac{\sqrt{2}}{2}
\end{bmatrix},
\]

are examples of unit vectors.
Example

\[ \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \] is not a unit vector, since \( ||\vec{v}|| = \sqrt{14}. \)
Example

\[ \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \] is not a unit vector, since \( \|\vec{v}\| = \sqrt{14}. \) However,

\[ \vec{u} = \frac{1}{\sqrt{14}} \vec{v} = \begin{bmatrix} \frac{-1}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \end{bmatrix} \]

is a unit vector in the same direction as \( \vec{v}, \)
Example

\[ \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \] is not a unit vector, since \( ||\vec{v}|| = \sqrt{14} \). However,

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is a unit vector in the same direction as \( \vec{v} \), i.e.,

\[ ||\vec{u}|| = \frac{1}{\sqrt{14}} ||\vec{v}|| = \frac{1}{\sqrt{14}} \sqrt{14} = 1. \]
Example (§4.1 Example 4)

If $\vec{v} \neq \vec{0}$, then

$$\frac{1}{||\vec{v}||} \vec{v}$$

is a unit vector in the same direction as $\vec{v}$. 
Example

Find the point, $M$, that is midway between $P_1(-1, -4, 3)$ and $P_2(5, 0, -3)$.
Example

Find the point, \( M \), that is midway between \( P_1(-1, -4, 3) \) and \( P_2(5, 0, -3) \).

\[
\overrightarrow{PM} = \overrightarrow{0P_1} + \frac{1}{2} \overrightarrow{P_1P_2} = \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}.
\]

Therefore \( M = (2, -2, 0) \).
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Find the point, \( M \), that is midway between \( P_1(-1, -4, 3) \) and \( P_2(5, 0, -3) \).

\[
\overrightarrow{0M} = \overrightarrow{0P_1} + \overrightarrow{P_1M} = \overrightarrow{0P_1} + \frac{1}{2} \overrightarrow{P_1P_2} = \begin{bmatrix} -1 \\ -4 \\ 3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 6 \\ 4 \\ -6 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}.
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\overrightarrow{0M} = \overrightarrow{0P_1} + \overrightarrow{P_1M} = \overrightarrow{0P_1} + \frac{1}{2} \overrightarrow{P_1P_2} = \\
\begin{bmatrix}
-1 \\
-4 \\
3
\end{bmatrix} + \frac{1}{2} \begin{bmatrix}
6 \\
4 \\
-6
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-1 \\
-4 \\
3
\end{bmatrix} + \begin{bmatrix}
3 \\
2 \\
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2 \\
2 \\
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\end{bmatrix}.
\]

Therefore \( M = M(2, -2, 0) \).
Example (§4.1 Exercise 15)

Find the two points trisecting the segment between $P(2, 3, 5)$ and $Q(8, -6, 2)$. 

Since $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -9 \\ -3 \end{pmatrix}$, $\overrightarrow{0A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$, and $\overrightarrow{0B} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ -6 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$.

Therefore, the two points are $A(4, 0, 4)$ and $B(6, -3, 3)$. 
Example (§4.1 Exercise 15)

Find the two points trisecting the segment between $P(2, 3, 5)$ and $Q(8, -6, 2)$.

The points are $A(4, 0, 4)$ and $B(6, -3, 3)$.

\[ \overrightarrow{0A} = \overrightarrow{OP} + \frac{1}{3} \overrightarrow{PQ} \text{ and } \overrightarrow{OB} = \overrightarrow{OP} + \frac{2}{3} \overrightarrow{PQ}. \]
Example (§4.1 Exercise 15)

Find the two points trisecting the segment between $P(2, 3, 5)$ and $Q(8, -6, 2)$.

Since $\overrightarrow{PQ} = \begin{bmatrix} 6 \\ -9 \\ -3 \end{bmatrix}$, 

$$\overrightarrow{0A} = \overrightarrow{0P} + \frac{1}{3} \overrightarrow{PQ} \quad \text{and} \quad \overrightarrow{0B} = \overrightarrow{0P} + \frac{2}{3} \overrightarrow{PQ}.$$ 

Since $\overrightarrow{PQ} = \begin{bmatrix} 6 \\ -9 \\ -3 \end{bmatrix}$, 

$$\overrightarrow{0A} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}, \quad \text{and} \quad \overrightarrow{0B} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 \\ -6 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}. $$
Example (§4.1 Exercise 15)

Find the two points trisecting the segment between $P(2, 3, 5)$ and $Q(8, -6, 2)$.

\[ \overrightarrow{0A} = \overrightarrow{OP} + \frac{1}{3} \overrightarrow{PQ} \text{ and } \overrightarrow{0B} = \overrightarrow{OP} + \frac{2}{3} \overrightarrow{PQ}. \] Since \( \overrightarrow{PQ} = \begin{bmatrix} 6 \\ -9 \\ -3 \end{bmatrix}, \)

\[ \overrightarrow{0A} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}, \] and
\[ \overrightarrow{0B} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 \\ -6 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}. \]

Therefore, the two points are $A(4, 0, 4)$ and $B(6, -3, 3)$. 
Example (§4.1 Example 6)

Let $ABCD$ be an arbitrary quadrilateral. Show that the midpoints of the four sides of $ABCD$ are the vertices of a parallelogram.

Let $M_1$ denote the midpoint of $\overrightarrow{AB}$, 
$M_2$ the midpoint of $\overrightarrow{BC}$, 
$M_3$ the midpoint of $\overrightarrow{CD}$, and 
$M_4$ the midpoint of $\overrightarrow{DA}$.

It suffices to prove that $\overrightarrow{M_1M_2} = \overrightarrow{M_4M_3}$.
Definition

Two nonzero vectors are called parallel if and only if they have the same direction or opposite directions.
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Two nonzero vectors are called parallel if and only if they have the same direction or opposite directions.

Theorem (§4.1 Theorem 4)

Two nonzero vectors $\vec{v}$ and $\vec{w}$ are parallel if and only if one is a scalar multiple of the other.
Definition

Two nonzero vectors are called parallel if and only if they have the same direction or opposite directions.

Theorem (§4.1 Theorem 4)

Two nonzero vectors \( \vec{v} \) and \( \vec{w} \) are parallel if and only if one is a scalar multiple of the other.

In particular, if \( \vec{v} \) and \( \vec{w} \) are nonzero and have the same direction, then \( \vec{v} = \frac{\|\vec{v}\|}{\|\vec{w}\|} \vec{w} \); if \( \vec{v} \) and \( \vec{w} \) have opposite directions, then \( \vec{v} = -\frac{\|\vec{v}\|}{\|\vec{w}\|} \vec{w} \).
Definition

Two nonzero vectors are called parallel if and only if they have the same direction or opposite directions.

Theorem (§4.1 Theorem 4)

Two nonzero vectors $\vec{v}$ and $\vec{w}$ are parallel if and only if one is a scalar multiple of the other.

In particular, if $\vec{v}$ and $\vec{w}$ are nonzero and have the same direction, then $\vec{v} = \frac{||\vec{v}||}{||\vec{w}||} \vec{w}$; if $\vec{v}$ and $\vec{w}$ have opposite directions, then $\vec{v} = -\frac{||\vec{v}||}{||\vec{w}||} \vec{w}$.

Read §4.1 Example 7 yourselves – determining whether or not two vectors are parallel.
Equations of Lines

Let $L$ be a line, $P_0(x_0, y_0, z_0)$ a fixed point on $L$, $P(x, y, z)$ an arbitrary point on $L$, and $\vec{d} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ a direction vector for $L$, i.e., a vector parallel to $L$.

Then $\overrightarrow{0P} = \overrightarrow{0P_0} + \overrightarrow{P_0P}$, and $\overrightarrow{P_0P}$ is parallel to $\vec{d}$, so $\overrightarrow{P_0P} = t\vec{d}$ for some $t \in \mathbb{R}$. 
Equations of Lines

Let $L$ be a line, $P_0(x_0, y_0, z_0)$ a fixed point on $L$, $P(x, y, z)$ an arbitrary point on $L$, and $\vec{d} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ a direction vector for $L$, i.e., a vector parallel to $L$.

Then $0\vec{P} = 0\vec{P}_0 + \vec{P}_0\vec{P}$, and $\vec{P}_0\vec{P}$ is parallel to $\vec{d}$, so $\vec{P}_0\vec{P} = t\vec{d}$ for some $t \in \mathbb{R}$.

**Vector Equation of a Line**

$$0\vec{P} = 0\vec{P}_0 + t\vec{d}, \quad t \in \mathbb{R}.$$
Equations of Lines

Let $L$ be a line, $P_0(x_0, y_0, z_0)$ a fixed point on $L$, $P(x, y, z)$ an arbitrary point on $L$, and $\vec{d} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ a direction vector for $L$, i.e., a vector parallel to $L$.

Then $\overrightarrow{0P} = \overrightarrow{0P_0} + \overrightarrow{P_0P}$, and $\overrightarrow{P_0P}$ is parallel to $\vec{d}$, so $\overrightarrow{P_0P} = t\vec{d}$ for some $t \in \mathbb{R}$.

**Vector Equation of a Line**

$$\overrightarrow{0P} = \overrightarrow{0P_0} + t\vec{d}, \quad t \in \mathbb{R}.$$  

Notation in the text: $\vec{p} = \overrightarrow{0P}$, $\vec{p}_0 = \overrightarrow{0P_0}$, so $\vec{p} = \vec{p}_0 + t\vec{d}$.  

---

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In component form, this is written as

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} = \begin{bmatrix}
  x_0 \\
  y_0 \\
  z_0
\end{bmatrix} + t \begin{bmatrix}
  a \\
  b \\
  c
\end{bmatrix}, \quad t \in \mathbb{R}.
\]
In component form, this is written as

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix} + t \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}, \, t \in \mathbb{R}.
\]

**Parametric Equations of a Line**

\[
x = x_0 + ta \\
y = y_0 + tb, \quad t \in \mathbb{R}.
\]
\[
z = z_0 + tc
\]
In component form, this is written as

\[
\begin{bmatrix}
  x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
  x_0 \\
y_0 \\
z_0
\end{bmatrix} + t \begin{bmatrix}
  a \\
b \\
c
\end{bmatrix}, \quad t \in \mathbb{R}.
\]

**Parametric Equations of a Line**

\[
\begin{align*}
x &= x_0 + ta \\
y &= y_0 + tb, \quad t \in \mathbb{R} \\
z &= z_0 + tc
\end{align*}
\]

§4.1 Example 11 describes what happens with the parametric equations of a line in \(\mathbb{R}^2\).
Example (similar to §4.1 Example 8)

Find an equation for the line through two points $P(2, -1, 7)$ and $Q(-3, 4, 5)$. 

A direction vector for this line is $\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -5 \\ 5 \\ -2 \end{bmatrix}$.

Therefore, a vector equation of this line is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix} + t \begin{bmatrix} -5 \\ 5 \\ -2 \end{bmatrix}$. 
Example (similar to §4.1 Example 8)

Find an equation for the line through two points \( P(2, -1, 7) \) and \( Q(-3, 4, 5) \).

A direction vector for this line is

\[
\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -5 \\ 5 \\ -2 \end{bmatrix}.
\]
Example (similar to §4.1 Example 8)

Find an equation for the line through two points $P(2, -1, 7)$ and $Q(-3, 4, 5)$.

A direction vector for this line is

$$
\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -5 \\ 5 \\ -2 \end{bmatrix}.
$$

Therefore, a vector equation of this line is

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix} + t \begin{bmatrix} -5 \\ 5 \\ -2 \end{bmatrix}.
$$
Example (similar to §4.1 Example 9)

Find an equation for the line through $Q(4, -7, 1)$ and parallel to the line

\[
L : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}.
\]
Example (similar to §4.1 Example 9)

Find an equation for the line through $Q(4, -7, 1)$ and parallel to the line

$$L : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}.$$ 

The line has equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}, \ t \in \mathbb{R}.$$
Given two lines $L_1$ and $L_2$, find the point of intersection, if it exists.

$L_1: \begin{align*}
x &= 3 + t \\
y &= 1 - 2t \\
z &= 3 + 3t
\end{align*}
$L_2: \begin{align*}
x &= 4 + 2s \\
y &= 6 + 3s \\
z &= 1 + s
\end{align*}$

Lines $L_1$ and $L_2$ intersect if and only if there are values $s, t \in \mathbb{R}$ such that

$$3 + t = 4 + 2s \quad \text{and} \quad 1 - 2t = 6 + 3s$$

i.e., if and only if the system

$$\begin{align*}
2s - t &= -1 \\
3s + 2t &= -5 \\
s - 3t &= 2
\end{align*}$$

is consistent.
Example

Given two lines $L_1$ and $L_2$, find the point of intersection, if it exists.

$L_1: \begin{align*}
    x &= 3 + t \\
    y &= 1 - 2t \\
    z &= 3 + 3t
\end{align*}$

$L_2: \begin{align*}
    x &= 4 + 2s \\
    y &= 6 + 3s \\
    z &= 1 + s
\end{align*}$

Lines $L_1$ and $L_2$ intersect if and only if there are values $s, t \in \mathbb{R}$ such that

\[
\begin{align*}
    3 + t &= 4 + 2s \\
    1 - 2t &= 6 + 3s \\
    3 + 3t &= 1 + s
\end{align*}
\]
Example

Given two lines $L_1$ and $L_2$, find the point of intersection, if it exists.

$L_1$: $x = 3 + t$
$y = 1 - 2t$
$z = 3 + 3t$

$L_2$: $x = 4 + 2s$
$y = 6 + 3s$
$z = 1 + s$

Lines $L_1$ and $L_2$ intersect if and only if there are values $s, t \in \mathbb{R}$ such that

\[
\begin{align*}
3 + t &= 4 + 2s \\
1 - 2t &= 6 + 3s \\
3 + 3t &= 1 + s
\end{align*}
\]

i.e., if and only if the system

\[
\begin{align*}
2s - t &= -1 \\
3s + 2t &= -5 \\
s - 3t &= 2
\end{align*}
\]

is consistent.
Example (continued)

\[
\begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & -5 \\
1 & -3 & 2 \\
\end{bmatrix} \rightarrow \cdots \rightarrow \\
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

Note. You can check your work by setting \( s = -1 \) in the equation for \( L_2 \).
Example (continued)

\[
\begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & -5 \\
1 & -3 & 2
\end{bmatrix} \rightarrow \cdots \rightarrow \\
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}
\]

$L_1$ and $L_2$ intersect when $s = -1$ and $t = -1$. 

Note. You can check your work by setting $s = -1$ in the equation for $L_2$. 

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Example (continued)

\[
\begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & -5 \\
1 & -3 & 2
\end{bmatrix}
\rightarrow \cdots \rightarrow
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}
\]

$L_1$ and $L_2$ intersect when $s = -1$ and $t = -1$.

Using the equation for $L_1$ and setting $t = -1$, the point of intersection is

\[P(3 + (-1), 1 - 2(-1), 3 + 3(-1)) = P(2, 3, 0).\]
Example (continued)

\[
\begin{bmatrix}
2 & -1 & -1 \\
3 & 2 & -5 \\
1 & -3 & 2
\end{bmatrix} \rightarrow \cdots \rightarrow \\
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}
\]

$L_1$ and $L_2$ intersect when $s = -1$ and $t = -1$.

Using the equation for $L_1$ and setting $t = -1$, the point of intersection is

\[P(3 + (-1), 1 - 2(-1), 3 + 3(-1)) = P(2, 3, 0).\]

Note. You can check your work by setting $s = -1$ in the equation for $L_2$. 
Example (§4.1 Exercise 22(g))

Find equations for the lines through $P(1, 0, 1)$ that meet the line

$$L : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

at points distance three from $P_0(1, 2, 0)$.
Example (§4.1 Exercise 22(g))

Find equations for the lines through $P(1, 0, 1)$ that meet the line

$$L : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

at points distance three from $P_0(1, 2, 0)$.

Find points $Q_1$ and $Q_2$ on $L$ that are distance three from $P_0$, and then find equations for the lines through $P$ and $Q_1$, and through $P$ and $Q_2$. 

\[\]
First, $||\vec{d}|| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$, so $\overrightarrow{0Q_1} = \overrightarrow{0P_0} + 1\vec{d}$, and $\overrightarrow{0Q_2} = \overrightarrow{0P_0} - 1\vec{d}$.

\[ \vec{d} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \]
Example (continued)

\[ \mathbf{d} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \]

First, \( ||\mathbf{d}|| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3 \), so

\[ \overrightarrow{0Q_1} = \overrightarrow{0P_0} + 1\mathbf{d}, \quad \text{and} \quad \overrightarrow{0Q_2} = \overrightarrow{0P_0} - 1\mathbf{d}. \]
First, $||\vec{d}|| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$, so

$0\vec{Q}_1 = 0\vec{P}_0 + 1\vec{d}$, and $0\vec{Q}_2 = 0\vec{P}_0 - 1\vec{d}$.

$0\vec{Q}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $0\vec{Q}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix},$

so $Q_1 = Q_1(3, 1, 2)$ and $Q_2 = Q_2(-1, 3, -2)$. 
Example (continued)

Equations for the lines:

- the line through \( P(1, 0, 1) \) and \( Q_1(3, 1, 2) \)

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix} = \overrightarrow{0P} + \overrightarrow{PQ_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}
\]

- the line through \( P(1, 0, 1) \) and \( Q_2(-1, 3, -2) \)

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix} = \overrightarrow{0P} + \overrightarrow{PQ_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ -3 \end{bmatrix}
\]