18.02 ESG Exam 4
Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

Be careful about your orientations.

1. [30 points]
   Consider the ellipsoid E given by the equation $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{36} = 1$. Let $f(x, y, z) = \sqrt{2x^2 + 2y^2 + 1}$. Calculate the value of

$$\iint_E f(x, y, z) dS$$

   [Hint: parameterize E similarly to a sphere.]

2. [30 points] Let S be the part of the paraboloid $z = 1-x^2-y^2$ above the xy-plane, and let $\mathbf{F}(x, y, z) = (x^2+yz^4-\sin(z^2), x-4y+z, 4z+1)$. Compute

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS$$

by using the divergence theorem to reduce to a simpler surface integral.

3. [40 points]

Consider the torus of inner radius 3, outer radius 5, central axis the y-axis and with central plane the xz-plane. Let S be the part of this torus above the xy-plane, with outward pointing normal. Define $\mathbf{F}(x,y,z) = (xy + \cos(\frac{\pi}{2}e^{xyz})\sin(xy), x + \ln(x^2+y^2+z^2)z^{x^2+y^2}, x^2 + xy^4 - 2x\cos(y) - e^{5y-3\cos(x)})$. Compute

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS.$$