18.02 ESG Exam 3
Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [30+20EC points]
   Set up the integration in each part, but you do not need to evaluate the integral. You may use rectangular, cylindrical or spherical coordinates as you wish.

   (a) [15] Consider the region bounded by the two cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 1$. Find the average distance of a point in this region to the point on both cylinders with largest z-coordinate.

   (b) [15] Consider a solid circular torus of inner radius 3 and outer radius 5. If the density of the torus is given by the square of the distance to the center, find the mass of the torus.

   (c) [20 extra credit] Find the average distance between two points in the unit disk.

2. [20 points] In this problem, set up the integration AND evaluate. Integrate the function $x^2 + z^2$ over the region enclosed by the planes $z = 0$ and $z = 1$ and the cone $z^2 = x^2 + y^2$.

3. [25 points]

For each of the following vector fields, determine whether it is conservative. If it is, find a potential function.

(a) $\vec{F_1}(x, y, z) = (4x^3y + 3z, x^4 - z + 2, 3x + y)$.

(b) $\vec{F_2}(x, y, z) = (2x + y\cos(xy), x\cos(xy) - ze^{yz}, 2 - ye^{yz})$.

4. [25 points] Use any method we've learned to evaluate the following line integrals.

(a) $\int_{C_1} \vec{F_1}(x,y,z)\cdot d\vec{s}$ where $\vec{F_1}(x,y,z) = (4x^3y+3z, x^4-z+2, 3x+y)$ as in (3a) and C_1 is the portion of the parabola $x = y = z^2$ from $(1,1,-1)$ to $(1,1,1)$.

(b) $\int_{C_2} \vec{F_2}(x,y,z)\cdot d\vec{s}$ where $\vec{F_2}(x,y,z) = (2x + y\cos(xy), x\cos(xy) - ze^{yz}, 2 - ye^{yz})$ as in (3b) and C_2 is the twisted cubic given by $\gamma(t) = (t, t^2, t^3), 0 \le t \le 1$.