

## 18.02 ESG Exam 2&3 Makeup Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

Test 2 section

1. [30 points]  
Find the local maxima and minima of the function  $f(x, y, z) = xy^2 - 3xyz + 2z$  when restricted to the surface  $xy = z^2 - 1$ . Don't worry about classifying them.

2. [30 points]

Suppose  $f(x, y)$  is a function on the plane. You know that the tangent plane to  $f$  at the point  $(1, 2, 10)$  has the equation  $3x - 4y + z = 5$ , and that the gradient of  $f$  at the point  $(1, 2)$  has length 1. Find the directional derivative of  $f$  at  $(1, 2)$  in the direction pointing toward the origin.

3. [30 points]

Suppose  $z(x, y)$  is defined implicitly by the equation  $x^4 - y^4 - 4z^4 = 1$  near the point  $(3, 2, 2)$  and suppose that  $g(x, y, z)$  is a function defined on all of  $\mathbb{R}^3$ . Set  $h(x, y) = g(x, y, z(x, y))$ . Suppose that  $\nabla h(3, 2) = (1, 2)$ . Find the directional derivative of  $g$  in the direction of the vector  $(32, 28, 4)$ .

Test 3 Section

4. [40 points]

Set up integrals for the following problems but do not evaluate.

- (a) [20] Find the mass of the region enclosed by the two surfaces  $x = 5 - z^2$  and  $x = 14 - y^2$  if the density of the region is given by the square of the distance from the  $xy$ -plane.
- (b) [20] Find the average value of the spherical coordinate angle  $\phi$  within a sphere of radius 5 centered at the point  $(3, 4, 7)$ .

5. [30 points]

Let  $\mathbf{F}(x, y) = (2xy + 2, x^2 - 1)$  and let  $C$  be portion of the graph of  $x^{2/3} + y^{2/3} = 1$  in the first quadrant, oriented so that the tangent vector points upward. Evaluate  $\int_C \mathbf{F}(x, y) \cdot \mathbf{T} ds$

(a) [15] Directly

(b) [15] Using the fundamental theorem of line integrals.

6. [20 points] Is the vector field

$$\mathbf{F}(x, y, z) = (2x \sin yz + 4xe^{x^2+y^2}, x^2z \cos yz + 4ye^{x^2+y^2}, x^2y \cos yz + \frac{1}{1 + (z + 2)^2})$$

conservative? If so, find a potential function.