1. [80 points]
   Let \( f(x, y) = 4xy - x^3y - xy^3 \).

   (a) [10] Compute all first and second order partial derivatives of \( f \).
(b) [30] Find and classify (as a local minimum, maximum or saddle point) all critical points of $f$. 
(c) [15] Suppose you now consider $f$ as a function on the disk $D$ of radius 2 around the origin. Write down the Lagrange multiplier equations for finding extrema of $f$ on the circle of radius 2 around the origin. Find the global minimum and global maximum value of $f$ on $D$. 
(d) [5] Find the gradient of $f$ at the point $(2, 2)$.
(e) [10] Find the tangent plane to $f$ at the point $(2, 2)$.
(f) [10] Find a direction $u$ such that $D_u(f) = -\frac{24}{5}$. 
2. [20 points]

Suppose that the variables $x, y, z$ satisfy an equation $g(x, y, z) = 0$. Assume the point $P(1, 1, 1)$ lies on this level surface of $g$ and that $\nabla g(1, 1, 1) = \langle -1, 1, 2 \rangle$. Let $f(x, y, z)$ be another function, and assume that $\nabla f(1, 1, 1) = \langle 1, 2, 1 \rangle$. Find the gradient of the function $w = f(x, y, z(x,y))$ of the two independent variables $x$ and $y$ at the point $x = 1, y = 1$. 