### 18.02 ESG Exam 2 Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [80 points]

Let $f(x, y)=4 x y-x^{3} y-x y^{3}$.
(a) [10] Compute all first and second order partial derivatives of $f$.
(b) [30] Find and classify (as a local minimum, maximum or saddle point) all critical points of $f$.
(c) [15] Suppose you now consider $f$ as a function on the disk $D$ of radius 2 around the origin. Write down the Lagrange multiplier equations for finding extrema of $f$ on the circle of radius 2 around the origin. Find the global minimum and global maximum value of $f$ on $D$.
(d) [5] Find the gradient of $f$ at the point $(2,2)$.
(e) [10] Find the tanget plane to $f$ at the point $(2,2)$.
(f) [10] Find a direction $\mathbf{u}$ such that $D_{\mathbf{u}}(f)=\frac{-24}{5}$.
2. [20 points]

Suppose that the variables $x, y, z$ satisfy an equation $g(x, y, z)=0$. Assume the point $P(1,1,1)$ lies on this level surface of $g$ and that $\nabla g(1,1,1)=<-1,1,2>$. Let $f(x, y, z)$ by another function, and assume that $\nabla f(1,1,1)=<1,2,1>$. Find the gradient of the function $w=f(x, y, z(x, y))$ of the two independent variables $x$ and $y$ at the point $x=1, y=1$.

