18.02 ESG Exam 1
Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [30 points]
   Consider the system of equations

$$\begin{aligned} 3x_1 + 5x_2 + 2x_3 &= 3 \\ 5x_1 + 3x_2 + x_3 &= 5 \\ 9x_1 + 6x_2 + 2x_3 &= 1 \end{aligned}$$

   (a) [5] Rewrite this system in the matrix form $A\mathbf{x} = \mathbf{b}$.

   (b) [10] Is A invertible? Justify your answer.

   (c) [15] Give all solutions to the system. Make sure to include your steps.

2. [25 points]

Consider the bounded region contained between the two paraboloids given by

$$z = a^2 - x^2 - y^2,$$
$$z = -b^2 + x^2 + y^2.$$

(a) [5] Write down the region (as on the first homework) enclosed by two surfaces, in cartesian coordinates.

(b) [5] Write down the equations of the surfaces in cylindrical coordinates.

(c) [5] Write down the region enclosed, in cylindrical coordinates.

(d) [5] Write down the equations of the surfaces in spherical coordinates.

(e) [5] Write down the region enclosed, in spherical coordinates.

3. [30 points]

Consider the curves defined parametrically by

$$\alpha_1(t) = (t^2 - 8t + 2, \frac{4}{3}t^3 + 2, 3t + 5)$$
$$\alpha_2(t) = (2t^2 - 3t + 1, 1 - t, 3t^2 + 4).$$

(a) [10] At what values t_1 and t_2 are the tangent vectors to the two curves perpindicular?

(b) [10] Determine the equation of the plane spanned by $\alpha_1'(t_1)$ and $\alpha_2'(t_1)$ passing through the point $(0, 0, 1)$ and the equation of the plane spanned by $\alpha_1'(t_2)$ and $\alpha_2'(t_2)$ passing through the point $(0, 0, 1)$.

(c) [10] Determine the parametric and symmetric equations of the line formed by intersecting these two planes.

4. [15 points]

Let S be the union of all lines passing through both the point $(0, 0, 1)$ and the curve in the xy-plane given by $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (Extra Credit: replace this by the curve $x^2y^2 = 1$). Parametrize S.