18.02 ESG Exam 1
Spring 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [30 points]
   Consider the system of equations

$$\begin{aligned} 2x_1 + 8x_2 - 4x_3 &= 4 \\ x_1 + 5x_2 \qquad\quad &= 3 \\ x_1 + \ \ x_2 - 8x_3 &= -1 \end{aligned}$$

   (a) [5] Rewrite this system in the matrix form $A\mathbf{x} = \mathbf{b}$.

   (b) [10] Is A invertible? Justify your answer.

   (c) [15] Give all solutions to the system. Make sure to include your steps.

2. [25 points]

Consider the ellipsoid given in rectangular coordinates by the equation

$$x^2 + y^2 + \frac{z^2}{c^2} = 1.$$

(a) [5] Write down the region (as on the first homework) enclosed by the ellipsoid, in cartesian coordinates.

(b) [5] Write down the equation of the surface in cylindrical coordinates.

(c) [5] Write down the region enclosed, in cylindrical coordinates.

(d) [5] Write down the equation of the ellipsoid in spherical coordinates.

(e) [5] Write down the region enclosed, in spherical coordinates.

3. [30 points]

Consider the curve defined parametrically by $\gamma(t) = (4e^{3t}, e^{2t}, 2e^{2t})$.

(a) [10] At what value t_0 is the tangent vector to γ at $\gamma(t_0)$ parallel to the plane $\mathbf{P}$ given by the equation $x + 2y - 7z = 17$?

(b) [10] Determine the parametric and symmetric equations of the line $\mathbf{L}$ tangent to γ at $\gamma(t_0)$.

(c) [10] Determine the parametric equation of the line passing through $\gamma(t_0)$, parallel to the plane $\mathbf{P}$ and perpendicular to the line $\mathbf{L}$.

4. [15 points]
Consider two helices around the z-axis of radius 1, separated by a $180°$ rotation. Parametrize the surface formed by connecting corresponding points in these helices with line segments.