18.02 Problem Set 8

(Due Wednesday, April 27, 11:59:59 PM)

Part I (63 points)

HAND IN ONLY THE UNDERLINED PROBLEMS
(The others are *some* suggested choices for more practice.)
EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Normal Form of Green's Theorem, Simply Connected Regions
Reading: EP §15.4 SN §§V3, V4, V5, V6
Exercises:
EP §15.4 21, 23, 26, 29, 35
SN §4G 5, 6
SN §6G 1

Stokes' Theorem
Reading: EP §15.7 SN §V13
Exercises:
EP §15.7 1, 3, 10, 16 SN §6F 5b

Part II (37 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1 () Let $f(x,y) = \frac{x^3y}{x^2+y^2} + I(x,y)$ and C be the circle in the xy-plane of radius 1 centered at the origin. Your objective is to compute

$$\oint_C xf(x,y)dx + yf(x,y)dy.$$

Unforetunately, the function I, though defined on the whole plane, is impossible to integrate. You would like to use Green's theorem to hopefully get rid of I, but there is a problem: $f(0,0)$ doesn't exist, and doing this wouldn't get rid of I anyway.

Being very insightful, you realize that Stokes' theorem could help. You can change your vector field, as long as it agrees with the old one on C, and use a surface in three dimensions for which C is the boundary. [Hint: Try changing f to $f(x,y,z) = \frac{x^3y}{x^2+y^2+z^2} + I(x - xz, y - yz)$ and using for a surface the cylinder of radius 1 topped by a disc in the $z = 1$ plane. You need to explain why all this works.]