18.02 Problem Set 7

(Due Wednesday, April 20, 11:59:59 PM)

Part I (77 points)

HAND IN ONLY THE UNDERLINED PROBLEMS
(The others are *some* suggested choices for more practice.)
EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Surface integrals
Reading: EP §§14.8, 15.5 SN §V9
Exercises:
EP §14.8 <u>9</u>, 12, 14, 16 EP §15.5 <u>3</u>, 5, <u>11</u>, <u>15</u>, 16, 22, <u>24</u>, <u>37</u> (Don't evaluate the integral)
SN §6B <u>1</u>, 2, 6, <u>7</u>, 12

Green's Theorem
Reading: EP §15.4
Exercises:
EP §15.4 <u>1</u>, 4, <u>9</u>, 18, <u>33</u>, 41

Part II (23 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1 (23) Consider the Klein bottle S, parameterized by

$$x = \cos u(\cos(\frac{u}{2})(\sqrt{2} + \cos v) + \sin(\frac{u}{2}) \sin v \cos v)$$
$$y = \sin u(\cos(\frac{u}{2})(\sqrt{2} + \cos v) + \sin(\frac{u}{2}) \sin v \cos v)$$
$$z = -\sin(\frac{u}{2})(\sqrt{2} + \cos v) + \cos(\frac{u}{2}) \sin v \cos v$$

where $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$. The Klein bottle is not orientable, so there is no continuous choice of normal vector and thus our definition of a flux integral is no longer well defined. But if we take the absolute value of $\vec{\mathbf{F}} \cdot \hat{\mathbf{n}}$ then we can integrate a vector field over this surface.

Let $\vec{\mathbf{F}}(x, y, z) = (0, 0, 1)$. Compute $\iint_S |\vec{\mathbf{F}} \cdot \hat{\mathbf{n}}| dS$.