18.02 Problem Set 4

(Due Tuesday, March 8, 11:59:59 PM)

Part I (90 points)

HAND IN ONLY THE UNDERLINED PROBLEMS
(The others are *some* suggested choices for more practice.)
EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Differentials, chain rule
Reading: EP §§13.6, 13.7 SN §N
Exercises:
EP §13.6 5, 8, 36, 40, 44
EP §13.7 5, 8, 9, 12, 23, 31, 48, 50, 51
SN §2C 3
SN §2E 2, 5

Gradient, directional derivatives
Reading: EP §§13.8
Exercises:
EP §13.8 2, 7, 16, 19, 21, 32, 46, 51, 60
SN §2D 1, 2abc, 3, 4

Lagrange multipliers
Reading: EP §13.9
Exercises:
EP §13.9 13, 22, 30, 43, 49, 62, 63
SN §2I 1ab, 2

Part II (10 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1 (10; 4, 6) Let $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2, f\colon (x,y,z) \mapsto (x^2+y^2, 2xyz)$ and let $g\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^3, g\colon (u,v) \mapsto (u-1, uv, v)$
We computed in Problem Set 1 that $g(f(x,y,z)) = (x^2+y^2-1, 2xyz(x^2+y^2), 2xyz)$.
a) Compute the total derivative (in matrix form) of $g \circ f$ at the point (a,b,c) directly.
b) Compute it using the chain rule.