

**18.02      Problem Set 3**  
**(Due Tuesday, March 1, 11:59:59 PM)**

**Part I    (75 points)**

HAND IN ONLY THE UNDERLINED PROBLEMS

(The others are *some* suggested choices for more practice.)

EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

**Partial derivatives, differentiability, total derivative**

Reading: EP §§13.2, 13.3, 13.4, end of 13.7

Exercises:

EP §13.2 38, 53, 54, 55, 56, 57, 58

EP §13.3 41, 51

EP §13.4 15, 16, 24, 41, 56, 58, 65, 68, 73

**Tangent Planes, linear approximation**

Reading: EP §§13.6, 13.8 SN §TA

Exercises:

EP §13.4 39

EP §13.8 31, 34

**Min-max problems, compact sets, least squares**

Reading: EP §13.5, SN §LS

Exercises:

EP §13.5 11, 30, 38, 49, 59, 61, 68

SN §2G 1c

**Second derivative test, boundaries, infinity**

Reading: EP §13.10, SN §SD

Exercises:

EP §13.5 25, 57

EP §13.10 5, 7, 20, 21, 25, 33

SN §2H 7

## Part II (25 points)

**Directions:** Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

**Problem 1** (18 + 10 extra credit) This problem is designed to give you a feel for how you might use this material in later classes here at MIT. This example, and the handout on modelling data using least squares, is taken from 12.410J (Observational Techniques in Optical Astronomy). The numbers involved are changed to ease computation, though you're still welcome to use Mathematica or Maple.

You're attempting to calibrate a spectrograph using an argon lamp. The spectrograph splits incoming light into its different wavelength components, which are then imaged on a CCD camera. But you don't know which wavelength corresponds to which column on the image. This is where the argon lamp comes in. Since argon emits radiation at a certain set of well defined wavelengths, you can match the wavelengths with the columns and use this calibration when you're taking images of stars or planets.

You have the following data points:

$$(7000\text{\AA}, 640 \pm 2 \text{ pixels})$$

$$(7500\text{\AA}, 530 \pm 4 \text{ pixels})$$

$$(8000\text{\AA}, 410 \pm 8 \text{ pixels})$$

$$(9000\text{\AA}, 200 \pm 3 \text{ pixels})$$

$$(9500\text{\AA}, 130 \pm 12 \text{ pixels}).$$

Fit a quadratic of the form  $y = a(x - x_0)^2 + b(x - x_0) + c$  to this data set, where  $y$  is the column (in pixels) corresponding to the wavelength  $x$ . Choose  $x_0$  to be the average of the wavelengths of the given data points. The coefficients  $a, b$  and  $c$  are the variables that you are changing in order to optimize the fit. You may ignore the errors on the column measurements and assign all the points the same weight (though for extra credit read the material in the handout about weights and do the fitting with the given standard deviations instead of weighting all points the same. Note that  $\pm 2$  means that  $\sigma = 2$  is the standard deviation).

**Problem 2** (7) Consider the function

$$f(x, y, z) = 2 \frac{x^2 + y^2}{x^2 + y^2 + 1} + e^{-(y^2 + z^2)}$$

defined on all of  $\mathbb{R}^3$ . Does this function have a global maximum? Justify your answer. If it does, find the maximum value.