Practice Questions for Final
18.02 ESG Spring 2005

1. Describe the set of vectors $mathbfv$ such that $\frac{\mathbf{v}}{|\mathbf{v}|} \cdot (1,1,1) = 1$.

2. At each point of the curve C parameterized by $\gamma(t) = (t^2+4t-2, t^3+5, \sin t - 4t^2)$, attach the vector $\gamma'(t) \times \gamma''(t)$. Parameterize the curve traced out by the heads of these vectors.

3. Let L be the line passing through the point $(1,4,5)$ in the direction $(2,8,1)$, and let P be the plane $3x+5y-z=2$. Let L' be the projection of L onto P. Find the parametric and symmetric equations for the line lying in the plane formed by L and L' bisecting the smaller (ie less than $\frac{\pi}{2}$) angle between them.

4. CHALLENGE PROBLEM (I think this problem is cool. By no means are you expected to know how to do this.) Suppose A and B are 2×2 matrices, and $A, A+B, A+2B, A+3B$, and $A+4B$ are all invertible integer matrices (they are invertible, with integer entries, and their inverses have integer entries. Prove that $A+5B$ is also an invertible integer matrix.

5. Consider the map $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^5$ given by $f(x,y,z) = (x-y, x+z, y-2z, x+y, z)$. What is the dimension of the image of f?

6. Find the critical points of the function $f(x,y,z) = x^2y - yz$ when constrained to the surface $x^2 + yz + \frac{y^2}{2} = 4$.

7. Let $f(x,y) = (x^2y, x-y^3+2, x^3y+xy^3)$. Compute the matrix derivative of f at the point $(1,2,4)$. What is the geometric meaning of this derivative?

8. Find and classify all critical points of $f(x,y,z) = xy + x^5 - y^3$.

9. Explain why the gradient is always perpindicular to any level surface.

10. Find the intersection of the tangent plane to $f(x,y) = 4x^3 - 3y^2$ at the point $(2,4)$ with the xy-plane.

11. Suppose $f(x,y,z) = xyz^2+2x-yz$. Compute the directional derivative of f at the point $(1,1,1)$ in the direction $(1,2,-1)$.

12. Let R be the region bounded above by the surface $z = 16-x^4-2x^2y^2-y^4$ and below by the lower half of the sphere of radius 2 centered at the origin. If the density of the region is given by $\delta(x,y,z) = x^2+y^2$, find the center of mass of R.

13. Let R be the region contained within both ellipsoids $x^2+y^2+4z^2 = 4$ and $4x^2+36y^2+9z^2 = 36$. Write an integral that gives the volume of R but do not evaluate it. [This one is a bit annoying]

14. Suppose A and B are 3×3 matrices, and

$$AB = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 8 \\ 9 & 0 & 2 \end{pmatrix}$$

Is it possible to tell if A is invertible? If so, is it invertible or not, and why?

15. Suppose that you are blowing up a cylindrical balloon, with your mouth covering one end of the cylinder, so that the surface of the balloon consists of the sides of the cylinder together with one circular endcap. Being in a hurry, you want the volume to increase at a rate of at least 160 cubic centimeters per second. Unforetunately, the balloon is made of a strange material that breaks if it travels faster than 4 centimeters per second. This condition applies to all sides of the balloon. In order to satisfy the desired rate of increase of volume, the surface area of the balloon must be above a certain threshold. What is this threshold?

16. Let C be the triangle with vertices $(1,5),(2,3)$ and $(3,4)$, oriented counterclockwise. Let $\mathbf{F} = (x+3y^2, 4y-2)$. Compute the flux integral $\oint_C \mathbf{F}\cdot\mathbf{n}\,ds$.