18.02ρ ESG Exam 4
Fall 2005

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [20 points]
   Let S be a sphere of radius 1, and let the density be equal to the distance *along the sphere* from the north pole of the sphere. Find the mass.

2. [45 points] For each of the following integrals, use either Green's Theorem, the Divergence Theorem or Stokes' Theorem to write the given integral as an iterated integral with a different number of integral signs (by iterated integral I mean a triple integral, a double integral or a single integral: ie, you need to transform surface integrals and line integrals to double and single integrals and put appropriate limits on your integrals). **Write the name of the theorem that you're using**.

(a) [15] Let C be the curve given parametrically by

$$\gamma(t) = ((t^3 - \frac{\pi^2 t}{9})^2 \cos t, (t^3 - \frac{\pi^2 t}{9})^2 \sin t, 0)$$

where t runs from $-\frac{pi}{3}$ to $\frac{\pi}{3}$. Let

$$\mathbf{F}(x, y, z) = (xy, x^2 + yz, x + y + 3z).$$

Modify the integral

$$\int_C \mathbf{F} \cdot d\mathbf{s}.$$

(b) [15] Let S be the half of the unit sphere that lies above the xy-plane, with upward pointing unit normal.
Let $\mathbf{F}(x, y, z) = (2y, 0, 2x)$. Modify the integral

$$\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dS.$$

(c) [15] Let S be the surface obtained by rotating the circle

$$(x-1)^2 + z^2 = 1$$

about the z-axis, oriented outward.
Let $\mathbf{F}(x, y, z) = (x^3 - x, y - y^2 z, yz^2)$. Modify the integral

$$\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dS.$$

3. [35 points]

Let S be the unit sphere centered at the origin with outward pointing normal and let $\mathbf{F}(x, y, z) = (x, 0, z)$.

(a) [10] Find the flux of $\mathbf{F}$ through S directly.

(b) [5] Use the divergence theorem to check your answer.

(c) [10] Now consider slicing the sphere with planes $z = a$ as a ranges from -1 to 1. Within each plane, use the normal form of Green's theorem to find the flux of F out of the circle *in that plane* (as a function of a).

(d) [10] Integrate your result from part (c) over the different values of a. Why does this result not agree with your answers in parts (a) and (b)?