$\begin{array}{c} \mathbf{18.02}\rho \ \mathbf{ESG} \ \mathbf{Exam} \ \mathbf{3} \\ \mathbf{Fall} \ \mathbf{2005} \end{array}$

Write your name in the top right corner of this page. Work in the space provided or on the backs of pages. You are allowed one page of notes and the use of a calculator, but you must show your work to get full credit and no other aids are allowed.

1. [20 points]

Set up the following integration, but do not evaluate the integral. You may use rectangular, cylindrical or spherical coordinates as you wish.

Integrate the square of the distance from the origin over the region contained within the surface given parametrically by the equations

$$x = (5 + \sin^2(4u)\cos(2v))\cos(v)\sin(u)$$

$$y = (5 + \sin^2(4u)\cos(2v))\sin(v)\sin(u)$$

$$z = (5 + \sin^2(4u)\cos(2v))\cos(u)$$

$$0 < v < 2\pi \text{ and } 0 < u < \pi$$

2. [30 points] Consider the region bounded by the surface

$$z = (3x + 5y - 1)^2 + (2x + 4y - 1)^2$$

and the plane

$$z = 10x + 18y + 2.$$

- (a) [15] Set up an integral to find the volume of this region. [Hint: 10x + 18y = 2(3x + 5y - 1) + 2(2x + 4y - 1)]
- (b) [15] Evaluate the integral you found in the first part.

3. [15 points]

Consider the vector field

$$\vec{F}(x,y,z) = (2xy + z, x^2 + 3y^2z, y^3 + x - 4z^3).$$

- (a) [5] Is \vec{F} conservative? Why or why not?
- (b) [10] If \vec{F} is conservative, find a potential function. If not, find a closed path C such that $\int_C \vec{F} \cdot d\vec{s} \neq 0$.

4. [20 points] Let C be the curve formed by intersecting the paraboloid

$$z = (x+y)^2 + (x+2y+1)^2$$

and the plane

$$x - y - z + 2 = 0,$$

oriented counterclockwise when viewed from above. Let f(x, y, z) be an arbitrary smooth function, and set

$$\vec{F}(x, y, z) = \nabla f + xy\mathbf{i} - xy\mathbf{j} - xy\mathbf{k}.$$

Is it possible to evaluate $\int_C \vec{F} \cdot d\vec{s}$ completely? If so, do so.

5. [15 points] Write an expression for the arclength of the Archimedian spiral $r = \theta$ between the origin and the point $(2\pi, 0)$. 6. [5 points EXTRA CREDIT] Let C be the twisted cubic (t, t^2, t^3) with $1 \le t \le 2$, and let

$$\vec{F}(x, y, z) = (y^{z} x^{y^{z}}, \ln(x) z y^{z-1} x^{y^{z}}, \ln(x) \ln(y) y^{z} x^{y^{z}}).$$

Evaluate $\int_C \vec{F}(x,y,z) \cdot d\vec{s}.$