Name ____________________

18.02ρ Exam 1

Work in the space provided (you may use the back). You may use a calculator and one page of notes. **Circle your answer!**

1) **(20 pts.)** Consider the three points $A = (3, 1, 4)$, $B = (6, 4, 4)$, $C = (3, 4, 1)$.

(a) (5 pts.) Find the area of the triangle formed by A, B, and C.

(b) (5 pts.) Find the angles of this triangle.

(c) (10 pts.) Find an equation for the plane containing A, B and C.

2) **(20 pts.)** Consider the helix given by the parametric equation $(\cos t,\, \sin t,\, t)$.

(a) (10 pts.) Find an equation of a general line tangent to the helix.

(b) (10 pts.) Find a parameterization of the curve traced out by the intersection of the xy-plane and the tangent lines described above.

3) **(20 pts.)** Let Q be the point $(1, 4, 3)$ and $\mathcal{P}$ be the plane given by the equation $2x + y - z = 4$.

(a) (10 pts.) Give parametric and symmetric equations of the line passing through Q and perpindicular to $\mathcal{P}$.

(b) (10 pts.) Find the perpindicular distance from Q to $\mathcal{P}$.

4) **(20 pts.)** Let $A = \begin{pmatrix} 1 & 2 & 5 \\ -1 & 1 & 4 \\ 2 & 5 & 12 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix}$. Consider the system of equations given by $A\mathbf{x} = \mathbf{b}$.

(a) (5 pts.) Is A invertible? Justify your answer.

(b) (15 pts.) Use Gaussian elimination to solve the system.

5) **(20 pts.)** Consider a pair of rods. The first rod of length a and is anchored at one end in the origin. It is revolving at a rate of one revolution per second in the xy-plane. At time $t = 0$ it points along the positive x-axis and it revolves counterclockwise. The second rod, of length b, is attached at one end to the other end of the first rod. It is also revolving at a rate of one revolution per second, but it's revolution is in the plane defined by the first rod and the z-axis. At time $t = 0$ it is pointing farther out along the positive x-axis and it's tip is moving in the positive z direction at time $t = 0$.

(a) (5 pts.) Find parametric equations for the end of the first rod.

(b) (10 pts.) Find parametric equations for the position of the end of the second rod in relation to the end of the first rod.

(c) (5 pts.) Find parametric equations for the position of the end of the second rod in relation to the origin.

Extra Credit

6) **(20 pts.)** Consider the lines given parametrically by the expressions

$$(0,\,0,\,1)+(1,\,1,\,-1)t,$$

$$(3,\,2,\,0)+(-1,\,2,\,-1)t,$$

$$(1,\,-1,\,1)+(1,\,0,\,-1)t,$$

and

$$(0,\,0,\,0)+(1,\,0,\,0)t.$$

Find a line intersecting all four of these lines. Give its parametric equations.

7) **(10 pts.)** Let L_1 be a line with equation $\mathbf{r}_1 + t\mathbf{v}_1$ and L_2 be a line with equation $\mathbf{r}_2 + t\mathbf{v}_2$. Assume that $\mathbf{v}_1$ and $\mathbf{v}_2$ are not parallel. Find an expression for the distance between L_1 and L_2 (ie the minimum distance between a point on L_1 and a point on L_2). Do not use components.

8) **(20 pts.)** The real projective plane is a nonorientable surface obtained by gluing a disc onto the edge of a Möbius strip. It can be embedded in $\mathbb{R}^4$ or $\mathbb{R}^5$. Find the equations of such an embedding.