18.02 Spring 2005

Test 1 Solutions

Problem 1

a)
\[
\begin{pmatrix}
2 & 8 & -4 \\
1 & 5 & 0 \\
1 & 1 & -8
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
= 
\begin{pmatrix}
4 \\
3 \\
-1
\end{pmatrix}.
\]

b) No: 
\[
det(A) = 2(5(-8) - 1(0)) - 8(1(-8) - 1(0)) + (-4)(1(1) - 1(5)) = 0.
\]

c)
\[
\begin{pmatrix}
2 & 8 & -4 & 4 \\
1 & 5 & 0 & 3 \\
1 & 1 & -8 & -1
\end{pmatrix}
\begin{pmatrix}
1 & 4 & -2 & 2 \\
0 & 1 & 2 & 1 \\
0 & -3 & -6 & -3 \\
1 & 0 & -10 & -2 \\
0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_2
\end{pmatrix}
= 
\begin{pmatrix}
-2 + 10t \\
2 - 2t \\
t
\end{pmatrix}.
\]

Thus the solution to the system is

Problem 2

a)
\[
-1 \leq x \leq 1,
\]
\[
-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2},
\]
\[
-c\sqrt{1-x^2-y^2} \leq z \leq c\sqrt{1-x^2-y^2}.
\]

b)
\[ r^2 + \frac{z^2}{c^2} = 1. \]

c) 
\[ 0 \leq \theta \leq 2\pi, \]
\[ 0 \leq r \leq 1, \]
\[ -c\sqrt{1 - r^2} \leq z \leq c\sqrt{1 - r^2}. \]

d) 
\[ \rho^2 \cos^2 \phi + \frac{\rho^2 \sin^2 \phi}{c^2} = 1. \]

e) 
\[ 0 \leq \theta \leq 2\pi, \]
\[ 0 \leq \phi \leq \pi, \]
\[ 0 \leq \rho \leq \frac{c}{\sqrt{\sin^2 \phi + c^2 \cos^2 \phi}}. \]

**Problem 3**

a) The tangent vector is given by differentiating \( \gamma \). It will be parallel to \( \mathbf{P} \) if it is orthogonal to \((1, 2, -7)\). Thus 
\[
(12e^{3t_0}, 2e^{2t_0}, 4e^{2t_0}) \cdot (1, 2, -7) = 12e^{3t_0} - 24e^{2t_0} = 12e^{2t_0}(e^{t_0} - 2),
\]
so \( t_0 = \ln 2 \).

b) Since \( \gamma(t_0) = (32, 4, 8) \) and \( \gamma'(t_0) = (96, 8, 16) \) the parametric equations are given by 
\[
v(s) = (32 + 96s, 4 + 8s, 8 + 16s)
\]
and the symmetric equations are given by 
\[
\frac{x - 32}{96} = \frac{y - 4}{8} = \frac{z - 8}{16}.
\]

c) It’s direction is given by \((1, 2, -7) \times (96, 8, 16) = (88, -688, -184)\) so the parametric equations are 
\[
v(s) = (32 + 88s, 4 - 688s, 8 - 184s).
\]

**Problem 4**

The two helices are given by the equations \((\cos t, \sin t, t)\) and \((-\cos t, \sin t, t)\). For each \( t \), one can draw the line segment between these two points. Call the parameter of the line segment \( s \). Then \((s \cos t, s \sin t, t)\) gives the line segment as \( s \) ranges from -1 to 1, so the surface is given parametrically by 
\[
(s \cos t, s \sin t, t)
\]
\[
-1 \leq s \leq 1
\]
\[
-\infty \leq t \leq \infty
\]