18.02 Spring 2005

Test 1 Solutions

Problem 1

a)

$$\begin{pmatrix} 2 & 8 & -4 \\ 1 & 5 & 0 \\ 1 & 1 & -8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}.$$

b) No: $\det(A) = 2(5(-8) - 1(0)) - 8(1(-8) - 1(0)) + (-4)(1(1) - 1(5)) = 0.$

c)

$$\left(\begin{array}{ccc|c} 2 & 8 & -4 & 4 \\ 1 & 5 & 0 & 3 \\ 1 & 1 & -8 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 1 & 5 & 0 & 3 \\ 1 & 1 & -8 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -3 & -6 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -10 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Thus the solution to the system is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 + 10t \\ 1 - 2t \\ t \end{pmatrix}$$

Problem 2

a)

$$-1 \leq x \leq 1,$$
$$-\sqrt{1 - x^2} \leq y \leq \sqrt{1 - x^2},$$
$$-c\sqrt{1 - x^2 - y^2} \leq z \leq c\sqrt{1 - x^2 - y^2}.$$

b)

$$r^2 + \frac{z^2}{c^2} = 1.$$

c)

$$0 \le \theta \le 2\pi,$$
$$0 \le r \le 1,$$
$$-c\sqrt{1-r^2} \le z \le c\sqrt{1-r^2}.$$

d)

$$\rho^2 \cos^2\phi + \frac{\rho^2 \sin^2\phi}{c^2} = 1.$$

e)

$$0 \le \theta \le 2\pi,$$
$$0 \le \phi \le \pi,$$
$$0 \le \rho \le \frac{c}{\sqrt{\sin^2\phi + c^2\cos^2\phi}}.$$

Problem 3

a) The tangent vector is given by differentiating γ. It will be parallel to $\mathbf{P}$ if it is orthogonal to $(1, 2, -7)$. Thus

$$(12e^{3t_0}, 2e^{2t_0}, 4e^{2t_0}) \cdot (1, 2, -7) = 12e^{3t_0} - 24e^{2t_0} = 12e^{2t_0}(e^{t_0} - 2),$$

so $t_0 = \ln 2$.
b) Since $\gamma(t_0) = (32, 4, 8)$ and $\gamma'(t_0) = (96, 8, 16)$ the parametric equations are given by

$$v(s) = (32 + 96s, 4 + 8s, 8 + 16s)$$

and the symmetric equations are given by

$$\frac{x-32}{96} = \frac{y-4}{8} = \frac{z-8}{16}.$$

c) It's direction is given by $(1, 2, -7) \times (96, 8, 16) = (88, -688, -184)$ so the parametric equations are

$$v(s) = (32 + 88s, 4 - 688s, 8 - 184s).$$

Problem 4

The two helices are given by the equations $(\cos t, \sin t, t)$ and $(-\cos t, \sin t, t)$. For each t, one can draw the line segment between these two points. Call the parameter of the line segment s. Then $(s\cos t, s\sin t, t)$ gives the line segment as s ranges from -1 to 1, so the surface is given parametrically by

$$(s\cos t, s\sin t, t)$$
$$-1 \le s \le 1$$
$$-\infty \le t \le \infty$$