Part I  (48 points)

HAND IN ONLY THE UNDERLINED PROBLEMS
(The others are some suggested choices for more practice.)

EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Normal Form of Green’s Theorem, Simply Connected Regions
Reading: EP §15.4 SN §§V3, V4, V5, V6
Exercises:
EP §15.4 21, 23, 26, 29
SN §4G 5, 6
SN §6G 1

Stokes’ Theorem
Reading: EP §15.7 SN §V13
Exercises:
EP §15.7 1, 3, 10, 16

Part II  (18 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1  () Let \( f(x, y) = \frac{x^3y}{x^2+y^2} + I(x, y) \) and \( C \) be the circle in the \( xy \)-plane of radius 1 centered at the origin. Your objective is to compute

\[
\oint_C xf(x, y)dx + yf(x, y)dy.
\]

Unfortunately, the function \( I \), though defined on the whole plane, is impossible to integrate. You would like to use Green’s theorem to hopefully get rid of \( I \), but there is a problem: \( f(0,0) \) doesn’t exist, and doing this wouldn’t get rid of \( I \) anyway.

Being very insightful, you realize that Stokes’ theorem could help. You can change your vector field, as long as it agrees with the old one on \( C \), and use a surface in three dimensions for which \( C \) is the boundary. [Hint: Try changing \( f \) to \( f(x, y, z) = \frac{x^3y}{x^2+y^2+z^2} + I(x-xz, y-yz) \) and using for a surface the cylinder of radius 1 topped by a disc in the \( z = 1 \) plane. You need to explain why all this works.]