18.02ρ Problem Set 4

(Due Tuesday, October 11, 11:59:59 PM)

Part I (50 points)

HAND IN ONLY THE UNDERLINED PROBLEMS
(The others are *some* suggested choices for more practice.)
EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Second derivative test, boundaries, infinity
Reading: EP §13.10, SN §SD
Exercises:
EP §13.5 25, <u>57</u>
EP §13.10 5, 7, <u>20</u>, 21, 25, <u>33</u>
SN §2H <u>7</u>

Differentials, chain rule
Reading: EP §§13.6, 13.7 SN §N
Exercises:
EP §13.6 5, <u>8</u>, 36, 40, 44
EP §13.7 5, <u>8</u>, 9, 12, 23, <u>31</u>, <u>48</u>, 50, 51
SN §2C <u>3</u>
SN §2E 2, <u>5</u>

Part II (16 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1 (8)
Consider the function

$$f(x, y, z) = 2\frac{x^2 + y^2}{x^2 + y^2 + 1} + e^{-(y^2+z^2)}$$

defined on all of $\mathbb{R}^3$. Does this function have a global maximum? Justify your answer. If it does, find the maximum value.

Problem 2 (8; 4, 4)
Let $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2, f\colon (x, y, z) \mapsto (x^2 + y^2, 2xyz)$ and let $g\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^3, g\colon (u, v) \mapsto (u - 1, uv, v)$
a) Compute the total derivative (in matrix form) of $g \circ f$ at the point (a, b, c) directly.
b) Compute it using the chain rule.